

Design Manual

Integrated Analysis and Design of Slab Systems

For SAFE™

Berkeley, California

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Chapter 1
Introduction

SAFE automates several slab and mat design tasks. Specifically, it integrates slab design moments across design strips and designs the required reinforcement; it checks slab punching shears around column supports and concentrated loads; and it designs beam flexural and shear reinforcements. The design procedures are described in the chapter entitled "SAFE Design Techniques" in the *Welcome to SAFE Manual*. The actual design algorithms vary based on the specific Design Code chosen by the user. This manual describes the algorithms used for the various codes.

It is noted that the design of reinforced concrete slabs is a complex subject and the Design Codes cover many aspects of this process. SAFE is a tool to help the user in this process. Only the aspects of design documented in this manual are automated by SAFE design. The user must check the results produced and address other aspects not covered by SAFE design.

Chapter 2
Design for ACI 318-05

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the American code ACI 318-05 (ACI 2005). For referencing to the pertinent sections of the ACI code in this chapter, a prefix "ACI" followed by the section number is used.

The design is based on user-specified loading combinations. However, the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Inch-Pound-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Inch-Pound-Second units unless otherwise noted.

2.1 Notations

This section identifies the various notations used in this chapter.

Table 2-1 List of Symbols Used in the ACI Code

A_g	Gross area of concrete, sq-in
A_s	Area of tension reinforcement, sq-in

Table 2-1 List of Symbols Used in the ACI Code

A'_s	Area of compression reinforcement, sq-in
$A_{s(\text{required})}$	Area of steel required for tension reinforcement, sq-in
A_v	Area of shear reinforcement, sq-in
A_v/s	Area of shear reinforcement per unit length of member, sq-in/in
a	Depth of compression block, in
a_b	Depth of compression block at balanced condition, in
$a_{\max}$	Maximum allowed depth of compression block, in
b	Width of member, in
b_f	Effective width of flange (T-Beam section), in
b_w	Width of web (T-Beam section), in
b_0	Perimeter of the punching critical section, in
b_1	Width of the punching critical section in the direction of bending, in
b_2	Width of the punching critical section perpendicular to the direction of bending, in
c	Depth to neutral axis, in
c_b	Depth to neutral axis at balanced conditions, in
d	Distance from compression face to tension reinforcement, in
d'	Concrete cover to center of reinforcing, in
d_s	Thickness of slab (T-Beam section), in
E_c	Modulus of elasticity of concrete, psi
E_s	Modulus of elasticity of reinforcement, assumed as 29,000,000 psi (ACI 8.5.2)

Table 2-1 List of Symbols Used in the ACI Code

f_c'	Specified compressive strength of concrete, psi
f_y	Specified yield strength of flexural reinforcement, psi
f_{ys}	Specified yield strength of shear reinforcement, psi
h	Overall depth of a section, in
M_u	Factored moment at section, lb-in
P_u	Factored axial load at section, lb
s	Spacing of the shear reinforcement along the length of the beam, in
V_c	Shear force resisted by concrete, lb
$V_{\max}$	Maximum permitted total factored shear force at a section, lb
V_u	Factored shear force at a section, lb
V_s	Shear force resisted by steel, lb
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
$\varepsilon_{c,\max}$	Maximum usable compression strain allowed in extreme concrete fiber, (0.003 in/in)
ε_s	Strain in reinforcing steel
$\varepsilon_{s,\min}$	Minimum tensile strain allowed in steel rebar at nominal strength for tension controlled behavior (0.005 in/in)
φ	Strength reduction factor
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear

2.2 Design Load Combinations

The design load combinations are the various combinations of the analysis cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (D), live load (L), pattern live load (PL), wind (W), earthquake (E) and snow (S) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be defined (ACI 9.2.1):

1.4D (ACI 9.2.1)

1.2D + 1.6L (ACI 9.2.1)

1.2D + 1.6(0.75 PL) (ACI 13.7.6.3)

0.9D ± 1.6W (ACI 9.2.1)
1.2D + 1.0L ± 1.6W

0.9D ± 1.0E (ACI 9.2.1)
1.2D + 1.0L ± 1.0E

The IBC 2006 basic load combinations (Section 1605.2.1) are the same. These are also the default design load combinations in SAFE whenever the ACI 318-05 code is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

2.3 Limits on Material Strength

The concrete strength f_c' should be 2500 psi and the upper limit of f_y should be 60,000 psi (ACI 3.5.3.2).

The code does not cover cases when the compression strength of concrete used in design is beyond the given limits or when the yield strength of steel used in design exceeds the given limits.

The program does not enforce any of these limits for flexure and shear design of beam and slabs and torsional design of beam. The specified strengths are used for design. The user is responsible for using the proper strength values while defining the materials.

2.4 Strength Reduction Factors

The strength reduction factors, φ, are applied on the specified strength to obtain the design strength provided by a member. The φ factors for flexure and shear are as follows:

$\varphi = 0.90$ for flexure (tension controlled) and (ACI 9.3.2.1)

$\varphi = 0.75$ for shear. (ACI 9.3.2.3)

The user is allowed to overwrite these values. However, caution is advised.

2.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on the beam moments, shear forces, load combination factors, and other criteria described in the following sections. The reinforcement requirements are calculated at the ends of the beam elements.

All beams are designed for major direction flexure and shear only. Effects resulting from any axial forces, minor direction bending and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement

2.5.1 Design Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the end of the beam elements. In designing the flexural reinforcement for the ma-

jor moment of a particular beam for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

2.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments produce bottom steel. In such cases, the beam may be designed as a Rectangular or a T-beam. Negative beam moments produce top steel. In such cases, the beam may be designed as a rectangular or inverted T-beam.

2.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 2-1 (ACI 10.2). Furthermore it is assumed that the net tensile strain of the reinforcing steel shall not be less than 0.005 (tension controlled; ACI 10.3.4). When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

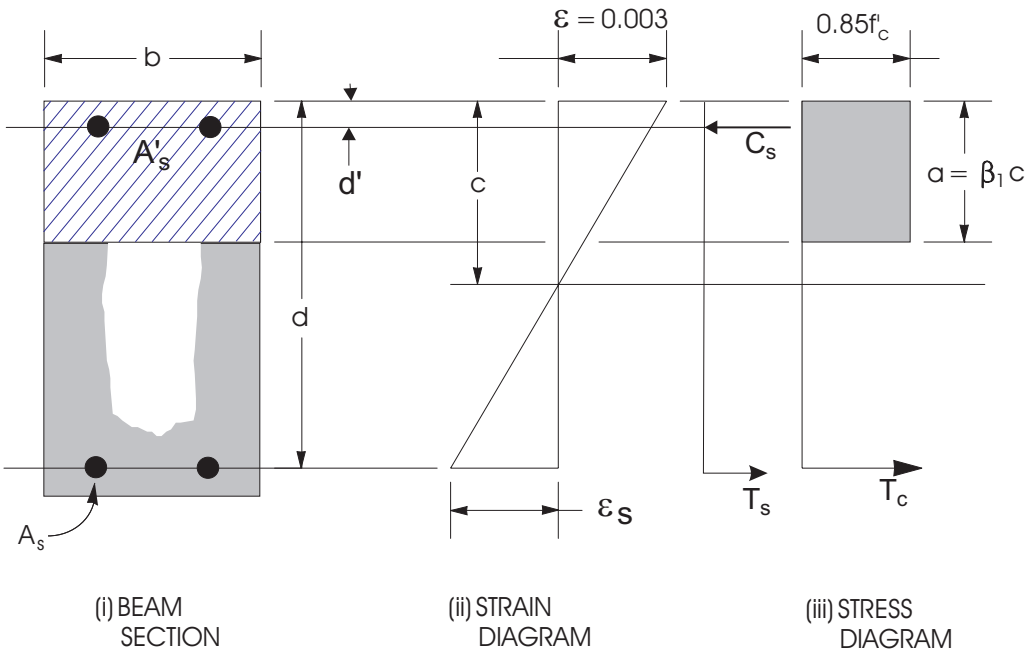


Figure 2-1 Rectangular Beam Design

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams) is summarized in the sections that follow. It is assumed that the design ultimate axial force does not exceed $\varphi(0.1f_c' A_g$; ACI 10.3.5); hence, all of the beams are designed for major direction flexure, shear, and torsion only.

2.5.1.2.1 Design for Rectangular Beam

In designing for a factored negative or positive moment, M_u (i.e., designing top or bottom steel), the depth of the compression block is given by a (see Figure 2-1), where,

$$a = d - \sqrt{d^2 - \frac{2|M_u|}{0.85 f'_c \varphi b}}, \qquad \text{(ACI 10.2)}$$

where, the value of φ is taken as that for a tension controlled section, which is 0.90 (ACI 9.3.2.1) in the above and the following equations.

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tensile steel tension shall not be less than ε_{smin}, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{max} = \frac{\varepsilon_{cmax}}{\varepsilon_{cmax} + \varepsilon_{smin}} \qquad \text{(ACI 10.2.2)}$$

where,

$$\varepsilon_{cmax} = 0.003 \qquad \text{(ACI 10.2.3)}$$

$$\varepsilon_{smin} = 0.005 \qquad \text{(ACI 10.3.4)}$$

The maximum allowable depth of the rectangular compression block, a_{max}, is given by

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 10.2.7.1)}$$

where β_1 is calculated as follows:

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right), \qquad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 10.2.7.3)}$$

- If $a \le a_{max}$ (ACI 10.3.4), the area of tensile steel reinforcement is then given by

  $$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)}.$$

  This steel is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

- If $a > a_{max}$, compression reinforcement is required (ACI 10.3.5) and is calculated as follows:

  The compressive force developed in concrete alone is given by

  $$C = 0.85 f_c' b a_{max}, \text{ and} \qquad \text{(ACI 10.2.7.1)}$$

  the moment resisted by concrete compression and tensile steel is

$$M_{uc} = C\left(d - \frac{a_{\max}}{2}\right)\varphi.$$

Therefore the moment resisted by compression steel and tensile steel is $M_{us} = M_u - M_{uc}$.

So the required compression steel is given by

$$A_s' = \frac{M_{us}}{\left(f'_s - 0.85f_c'\right)(d - d')\varphi}, \text{ where}$$

$$f_s' = E_s\varepsilon_{c\max}\left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_y. \qquad \text{(ACI 10.2.2, 10.2.3, and ACI 10.2.4)}$$

The required tensile steel for balancing the compression in concrete is

$$A_{s1} = \frac{M_{us}}{f_y\left[d - \frac{a_{\max}}{2}\right]\varphi}, \text{ and}$$

the tensile steel for balancing the compression in steel is given by

$$A_{s2} = \frac{M_{us}}{f_y(d - d')\varphi}.$$

Therefore, the total tensile reinforcement, $A_s = A_{s1} + A_{s2}$, and total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and vice versa if M_u is negative.

2.5.1.2.2 Design for T-Beam

(i) Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_u (i.e., designing top steel), the calculation of the steel area is exactly the same as above, i.e., no T-Beam data is to be used.

(ii) Flanged Beam Under Positive Moment

If $M_u > 0$, the depth of the compression block is given by

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.85 f'_c \varphi b_f}}, \qquad \text{(ACI 10.2)}$$

where, the value of φ is taken as that for a tension controlled section, which is 0.90 (ACI 9.3.2.1) in the above and the following equations.

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tensile steel tension shall not be less than ε_{smin}, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{max} = \frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}} \qquad \text{(ACI 10.2.2)}$$

where,

$$\varepsilon_{cmax} = 0.003 \qquad \text{(ACI 10.2.3)}$$

$$\varepsilon_{smin} = 0.005 \qquad \text{(ACI 10.3.4)}$$

The maximum allowable depth of the rectangular compression block, a_{max}, is given by

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 10.2.7.1)}$$

where β_1 is calculated as follows:

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right), \quad 0.65 \le \beta_1 \le 0.85. \qquad \text{(ACI 10.2.7.3)}$$

- If $a \le d_s$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular section design. However, in this case, the width of the beam is taken as b_f. Whether compression reinforcement is required depends on if $a > a_{max}$.
- If $a > d_s$, calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 2-2. C_f is given by

$$C_f = 0.85\, f_c'\, (b_f - b_w)\, \min(d_s,\, a_{\max}).$$

Therefore, $A_{s1} = \dfrac{C_f}{f_y}$ and the portion of M_u that is resisted by the flange is given by

$$M_{uf} = C_f \left(d - \frac{min\left(d_s, a_{\max}\right)}{2} \right) \varphi.$$

Again, the value for φ is 0.90. Therefore, the balance of the moment, M_u, to be carried by the web is given by

$$M_{uw} = M_u - M_{uf}.$$

The web is a rectangular section of dimensions b_w and d, for which the design depth of the compression block is recalculated as

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f'_c \varphi b_w}}. \qquad \text{(ACI 10.2)}$$

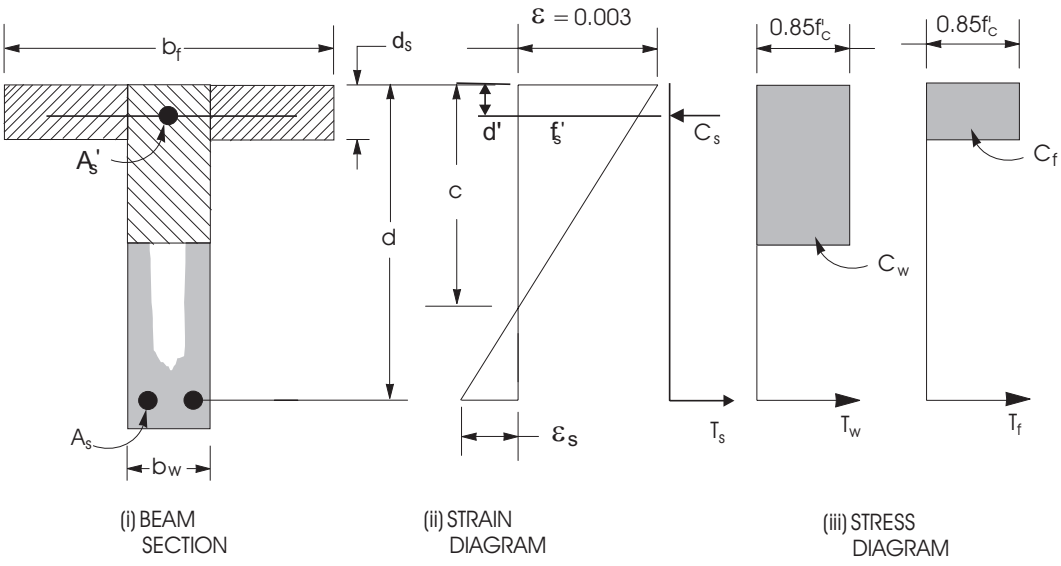


Figure 2-2 T-Beam Design

- If $a_1 \le a_{\max}$ (ACI 10.3.5), the area of tensile steel reinforcement is then given by

$$A_{s2} = \frac{M_{uw}}{\varphi f_y \left(d - \frac{a_1}{2} \right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2}.$$

This steel is to be placed at the bottom of the T-beam.

- If $a_1 > a_{max}$, compression reinforcement is required (ACI 10.3.5) and is calculated as follows:

The compressive force in the web concrete alone is given by

$$C = 0.85 f_c' b_w a_{\max}. \qquad \text{(ACI 10.2.7.1)}$$

Therefore, the moment resisted by the concrete web and tensile steel is

$$M_{uc} = C \left(d - \frac{a_{max}}{2} \right) \varphi, \text{ and}$$

the moment resisted by compression steel and tensile steel is

$$M_{us} = M_{uw} - M_{uc}.$$

Therefore, the compression steel is computed as

$$A_s' = \frac{M_{us}}{\left(f_s' - 0.85 f_c' \right) \left(d - d' \right) \varphi}, \text{ where}$$

$$f_s' = \varepsilon_s \varepsilon_{c\max} \left[\frac{c_{\max} - d'}{c_{\max}} \right] \le f_y. \qquad \text{(ACI 10.2.2, 10.2.3 and ACI 10.2.4)}$$

The tensile steel for balancing compression in web concrete is

$$A_{s2} = \frac{M_{uc}}{f_y\left[d - \frac{a_{max}}{2}\right]\varphi}, \text{ and}$$

the tensile steel for balancing compression in steel is

$$A_{s3} = \frac{M_{us}}{f_y\left(d - d'\right)\varphi}.$$

The total tensile reinforcement, $A_s = A_{s1} + A_{s2} + A_{s3}$, and total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top.

2.5.1.2.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel required in a beam section is given by the minimum of the two following limits:

$$A_s \geq \max\left\{\frac{3\sqrt{f_c'}}{f_y}\ b_w d \text{ and } \frac{200}{f_y} b_w d \quad \text{or} \right. \qquad \text{(ACI 10.5.1)}$$

$$A_s \geq \frac{4}{3}\ A_{s(\text{required})}. \qquad \text{(ACI 10.5.3)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{T - beam} \end{cases}$$

$$A_s' \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{T - beam} \end{cases}$$

2.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at two stations at the ends of each beam element. In designing the shear reinforcement

for a particular beam for a particular loading combination at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_u
- Determine the shear force, V_c, that can be resisted by the concrete
- Determine the reinforcement steel required to carry the balance

The following three sections describe in detail the algorithms associated with the preceding steps.

2.5.2.1 Determine Shear Force

In the design of the beam shear reinforcement of a concrete beam, the shear forces for a particular load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the corresponding load combination factors.

2.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as follows:

$$V_c = 2\sqrt{f_c'}\ b_w d. \qquad \text{(ACI 11.3.1.1)}$$

A limit is imposed on the value of $\sqrt{f_c'}$ as $\sqrt{f_c'} \le 100$. (ACI 11.1.2)

2.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum limit of

$$V_{max} = V_c + (8\sqrt{f_c'})\ b_w d. \quad \text{(ACI 11.5.7.9)}$$

Given V_u, V_c, and V_{max}, the required shear reinforcement is calculated as follows where, φ, the strength reduction factor, is 0.75 (ACI 9.3.2.3).

- If $V_u \le (V_c / 2)\ \varphi$,

$$\frac{A_v}{s} = 0, \qquad \text{(ACI 11.5.6.1)}$$

else if $(V_c / 2)\,\varphi < V_u \le \varphi V_{max}$,

$$\frac{A_v}{s} = \frac{\left(V_u - \varphi V_c\right)}{\varphi f_{ys} d}, \qquad \text{(ACI 11.5.7.1, 11.5.7.2)}$$

$$\frac{A_v}{s} \ge \max\left(\frac{0.75\sqrt{f_c'}}{f_y} b_w,\ \frac{50}{f_y} b_w\right) \qquad \text{(ACI 11.5.6.3)}$$

else if $V_u > \varphi V_{max}$,

a failure condition is declared. (ACI 11.5.7.9)

- If V_u exceeds the maximum permitted value of ϕV_{max}, the concrete section should be increased in size (ACI 11.5.7.9).

  Note that if torsion design is done and torsion rebar is needed, the equation given in ACI 11.5.6.3 does not need to be satisfied independently.

  The maximum of all the calculated A_v/s values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

  The beam shear reinforcement requirements displayed by the program are based purely upon shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

2.6 Slab Design

As is done in conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method (ACI 318-05) for reinforced concrete as described in the following sections. To learn

more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

2.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Design flexural reinforcement for the strip

These two steps, described in the following sections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combinations, is obtained and reported.

2.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

2.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip given the bending moment is identical to the design of rectangular beam sections described earlier (or to the T-beam if the slab is ribbed). Where the slab properties (depth, etc.) vary over the width of the strip, the program automatically designs slab

widths of each property separately for the bending moment to which they are subjected before summing up the reinforcement for the full width. Where openings occur the slab, width is adjusted accordingly.

2.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limits (ACI 7.12.2):

$$A_s \geq 0.0018\, bh\, \frac{60,000}{f_y} \qquad \text{(ACI 7.12.2.1)}$$

$$0.0014\, bh \leq A_s \leq 0.0020\, bh \qquad \text{(ACI 7.12.2.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

2.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the following sections.

2.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of *d/2* from the face of the support (ACI 11.12.1.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (ACI 11.12.1.3).

2.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$,

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \text{ and} \qquad \text{(ACI 13.5.3.2)}$$

$$\gamma_v = 1 - \gamma_f, \qquad \text{(ACI 13.5.3.1)}$$

where b_1 is the width of the critical section measured in the direction of the span, and b_2 is the width of the critical section measured in the direction perpendicular to the span.

2.6.2.3 Determination of Concrete Capacity

The concrete punching shear stress capacity is taken as the minimum of the following three limits:

$$v_c = min\begin{cases} \varphi\left(2+\dfrac{4}{\beta_c}\right)\sqrt{f_c'} \\ \varphi\left(2+\dfrac{a_s d}{b_o}\right)\sqrt{f_c'} \\ \varphi 4\sqrt{f_c'} \end{cases}$$

where, β_c is the ratio of the minimum to the maximum dimensions of the critical section, b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$a_s = \begin{cases} 40 & \text{for interior columns,} \\ 30 & \text{for edge columns, and} \\ 20 & \text{for corner columns.} \end{cases} \qquad \text{(ACI 11.12.2.1)}$$

A limit is imposed on the value of $\sqrt{f_c'}$ as $\sqrt{f_c'} \le 100$. (ACI 11.1.2)

2.6.2.4 Determination of Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by SAFE.

Chapter 3
Design for ACI 318-02

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the American code ACI 318-02 (ACI 2002). Various notations used in this chapter are listed in Table 3-1. For referencing to the pertinent sections of the ACI code in this chapter, a prefix "ACI" followed by the section number is used.

The design is based on user-specified loading combinations. However, the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Inch-Pound-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Inch-Pound-Second units unless otherwise noted.

3.1 Notations

This section identifies the various notations used in this chapter.

Table 3-1 List of Symbols Used in the ACI Code

A_g	Gross area of concrete, sq-in

Table 3-1 List of Symbols Used in the ACI Code

A_s	Area of tension reinforcement, sq-in
A'_s	Area of compression reinforcement, sq-in
$A_{s(\text{required})}$	Area of steel required for tension reinforcement, sq-in
A_v	Area of shear reinforcement, sq-in
A_v/s	Area of shear reinforcement per unit length of member, sq-in/in
a	Depth of compression block, in
a_b	Depth of compression block at balanced condition, in
$a_{\max}$	Maximum allowed depth of compression block, in
b	Width of member, in
b_f	Effective width of flange (T-Beam section), in
b_w	Width of web (T-Beam section), in
b_0	Perimeter of the punching critical section, in
b_1	Width of the punching critical section in the direction of bending, in
b_2	Width of the punching critical section perpendicular to the direction of bending, in
c	Depth to neutral axis, in
c_b	Depth to neutral axis at balanced conditions, in
d	Distance from compression face to tension reinforcement, in
d'	Concrete cover to center of reinforcing, in
d_s	Thickness of slab (T-Beam section), in
E_c	Modulus of elasticity of concrete, psi
E_s	Modulus of elasticity of reinforcement, assumed as 29,000,000 psi (ACI 8.5.2)

Table 3-1 List of Symbols Used in the ACI Code

f'_c	Specified compressive strength of concrete, psi
f_y	Specified yield strength of flexural reinforcement, psi
f_{ys}	Specified yield strength of shear reinforcement, psi
h	Overall depth of a section, in
M_u	Factored moment at section, lb-in
P_u	Factored axial load at section, lb
s	Spacing of the shear reinforcement along the length of the beam, in
V_c	Shear force resisted by concrete, lb
$V_{\max}$	Maximum permitted total factored shear force at a section, lb
V_u	Factored shear force at a section, lb
V_s	Shear force resisted by steel, lb
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
$\varepsilon_{c,\max}$	Maximum usable compression strain allowed in extreme concrete fiber, (0.003 in/in)
ε_s	Strain in reinforcing steel
$\varepsilon_{s,\min}$	Minimum tensile strain allowed in steel rebar at nominal strength for tension controlled behavior (0.005 in/in)
φ	Strength reduction factor
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear

3.2 Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (DL), live load (LL), pattern live load (PLL), wind (WL), and earthquake (EL) loads, and considering that wind and earthquake forces are reversible, the following load combinations must be considered (ACI 9.2.1).

1.4 DL
1.2 DL + 1.6 LL (ACI 9.2.1)

1.2 DL + 1.6 * 0.75 PLL (ACI 13.7.6.3)

0.9 DL ± 1.6 WL
1.2 DL + 1.0 LL ± 1.6 WL (ACI 9.2.1)

0.9 DL ± 1.0 EL
1.2 DL + 1.0 LL ± 1.0 EL (ACI 9.2.1)

The IBC 2003 basic load combinations (Section 1605.2.1) are the same. These are also the default design load combinations in SAFE whenever the ACI 318-02 code is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

3.3 Strength Reduction Factors

The strength reduction factors, φ, are applied on the specified strength to obtain the design strength provided by a member. The φ factors for flexure and shear are as follows:

$\varphi = 0.90$ for flexure (tension controlled) and (ACI 9.3.2.1)

$\varphi = 0.75$ for shear. (ACI 9.3.2.3)

The user is allowed to overwrite these values. However, caution is advised.

3.4 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on the beam moments, shear forces, load combination factors, and other criteria described in the following sections. The reinforcement requirements are calculated at the ends of the beam elements.

All beams are designed for major direction flexure and shear only. Effects resulting from any axial forces, minor direction bending, and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement

3.4.1 Design Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the end of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

3.4.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments produce bottom steel. In such cases, the beam may be designed as a Rectangular or a T-beam. Negative beam moments produce top

steel. In such cases, the beam may be designed as a rectangular or inverted T-beam.

3.4.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 3-1 (ACI 10.2). Furthermore, it is assumed that the net tensile strain of the reinforcing steel shall not be less than 0.005 (tension controlled) (ACI 10.3.4). When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the following sections. It is assumed that the design ultimate axial force does not exceed $\varphi(0.1f'_c A_g)$; ACI 10.3.5); hence all of the beams are designed for major direction flexure and shear only.

3.4.1.2.1 Design for Rectangular Beam

In designing for a factored negative or positive moment, M_u (i.e., designing top or bottom steel), the depth of the compression block is given by a (see Figure 3-1), where,

$$a = d - \sqrt{d^2 - \frac{2|M_u|}{0.85 f'_c \varphi b}}, \qquad \text{(ACI 10.2)}$$

where, the value of φ is taken as that for a tension controlled section, which is 0.90 (ACI 9.3.2.1) in the preceding and the following equations.

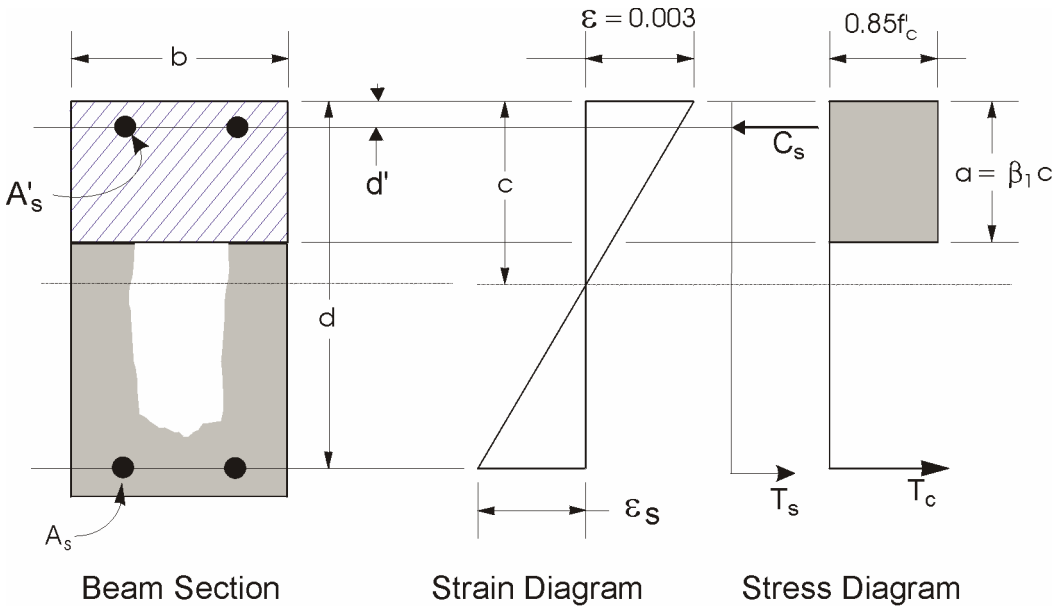


Figure 3-1 Rectangular Beam Design

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tensile steel tension shall not be less than ε_{smin}, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{\max} = \frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}} \qquad \text{(ACI 10.2.2)}$$

where,

$$\varepsilon_{c\max} = 0.003 \qquad \text{(ACI 10.2.3)}$$

$$\varepsilon_{s\min} = 0.005 \qquad \text{(ACI 10.3.4)}$$

The maximum allowable depth of the rectangular compression block, a_{max}, is given by

$$a_{\max} = \beta_1 c_{\max} \qquad \text{(ACI 10.2.7.1)}$$

where β_1 is calculated as follows:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \quad 0.65 \le \beta_1 \le 0.85. \qquad \text{(ACI 10.2.7.3)}$$

- If $a \le a_{max}$ (ACI 10.3.4), the area of tensile steel reinforcement is then given by

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)}.$$

This steel is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

- If $a > a_{max}$, compression reinforcement is required (ACI 10.3.5) and is calculated as follows:

The compressive force developed in concrete alone is given by

$C = 0.85 f'_c b a_{max}$, and (ACI 10.2.7.1)

the moment resisted by concrete compression and tensile steel is $M_{uc} = C\left(d - \frac{a_{max}}{2}\right)\varphi.$

Therefore the moment resisted by compression steel and tensile steel is

$M_{us} = M_u - M_{uc}.$

So the required compression steel is given by

$$A_s' = \frac{M_{us}}{\left(f_s' - 0.85 f_c'\right)\left(d - d'\right)\varphi}, \text{ where}$$

$$f_s' = E_s \varepsilon_{cmax}\left[\frac{c_{max} - d'}{c_{max}}\right] \le f_y. \quad \text{(ACI 10.2.2, 10.2.3, and ACI 10.2.4)}$$

The required tensile steel for balancing the compression in concrete is

$$A_{s1} = \frac{M_{us}}{f_y\left[d - \frac{a_{max}}{2}\right]\varphi}, \text{ and}$$

the tensile steel for balancing the compression in steel is given by

$$A_{s2} = \frac{M_{us}}{f_y (d - d')\varphi}.$$

Therefore, the total tensile reinforcement, $A_s = A_{s1} + A_{s2}$, and total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and vice versa if M_u is negative.

3.4.1.2.2 Design for T-Beam

(i) Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_u, (i.e., designing top steel), the calculation of the steel area is exactly the same as described previously, i.e., no T-Beam data is to be used.

(ii) Flanged Beam Under Positive Moment

If $M_u > 0$, the depth of the compression block is given by

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.85 f'_c \varphi b_f}}, \qquad \text{(ACI 10.2)}$$

where, the value of φ is taken as that for a tension controlled section, which is 0.90 (ACI 9.3.2.1) in the preceding and the following equations.

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tensile steel tension shall not be less than ε_{smin}, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{max} = \frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}} \qquad \text{(ACI 10.2.2)}$$

where,

$\varepsilon_{cmax} = 0.003$ (ACI 10.2.3)

$\varepsilon_{smin} = 0.005$ (ACI 10.3.4)

The maximum allowable depth of the rectangular compression block, a_{max}, is given by

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 10.2.7.1)}$$

where β_1 is calculated as follows:

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right), \qquad 0.65 \le \beta_1 \le 0.85. \quad \text{(ACI 10.2.7.3)}$$

- If $a \le d_s$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular section design. However, in this case, the width of the beam is taken as b_f. Whether compression reinforcement is required depends on whether $a > a_{max}$.
- If $a > d_s$, calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 3-2. C_f is given by

  $$C_f = 0.85 f_c' (b_f - b_w) \min(d_s, a_{max}).$$

  Therefore, $A_{s1} = \dfrac{C_f}{f_y}$ and the portion of M_u that is resisted by the flange is given by

  $$M_{uf} = C_f\left(d - \frac{\min(d_s, a_{max})}{2}\right)\varphi.$$

  Again, the value for φ is 0.90. Therefore, the balance of the moment, M_u to be carried by the web is given by

  $$M_{uw} = M_u - M_{uf}.$$

  The web is a rectangular section of dimensions b_w and d, for which the design depth of the compression block is recalculated as

  $$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f_c' \varphi b_w}}. \qquad \text{(ACI 10.2)}$$

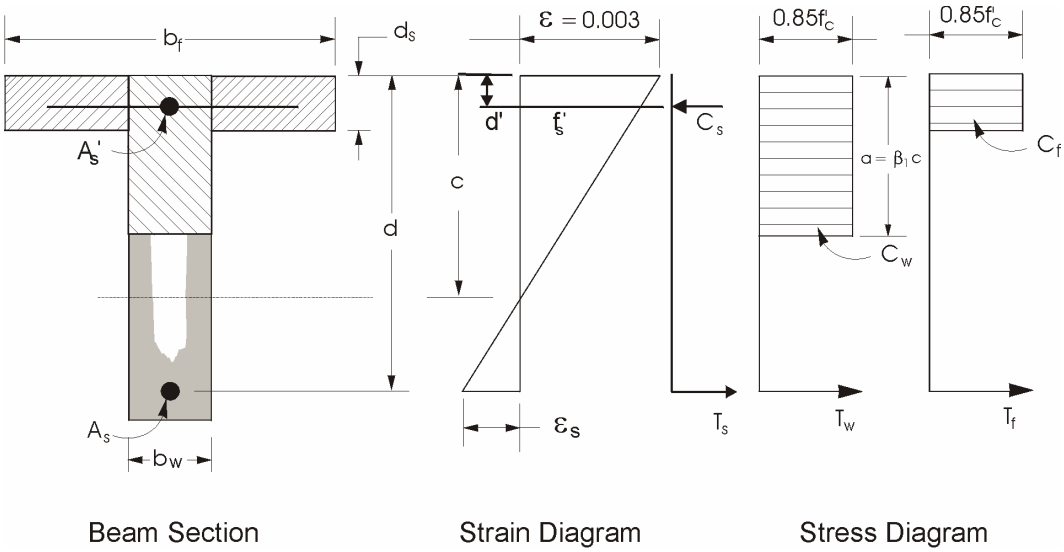


Figure 3-2 T-Beam Design

- If $a_1 \le a_{max}$ (ACI 10.3.5), the area of tensile steel reinforcement is then given by

$$A_{s2} = \frac{M_{uw}}{\varphi f_y \left(d - \frac{a_1}{2} \right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2}.$$

This steel is to be placed at the bottom of the T-beam.

- If $a_1 > a_{max}$, compression reinforcement is required (ACI 10.3.5) and is calculated as follows:

The compressive force in the web concrete alone is given by

$$C = 0.85 f_c' b_w a_{max}. \qquad \text{(ACI 10.2.7.1)}$$

Therefore the moment resisted by the concrete web and tensile steel is

$$M_{uc} = C \left(d - \frac{a_{max}}{2} \right) \varphi, \text{ and}$$

the moment resisted by compression steel and tensile steel is

$$M_{us} = M_{uw} - M_{uc}.$$

Therefore, the compression steel is computed as

$$A_s' = \frac{M_{us}}{\left(f_s' - 0.85 f_c'\right)\left(d - d'\right)\varphi}, \text{ where}$$

$$f_s' = \varepsilon_s \varepsilon_{cmax}\left[\frac{c_{max} - d'}{c_{max}}\right] \le f_y. \qquad \text{(ACI 10.2.2, 10.2.3 and ACI 10.2.4)}$$

The tensile steel for balancing compression in web concrete is

$$A_{s2} = \frac{M_{uc}}{f_y\left[d - \frac{a_{max}}{2}\right]\varphi}, \text{ and}$$

the tensile steel for balancing compression in steel is

$$A_{s3} = \frac{M_{us}}{f_y\left(d - d'\right)\varphi}.$$

The total tensile reinforcement, $A_s = A_{s1} + A_{s2} + A_{s3}$, and total compression reinforcement is A_s'. A_s is to be placed at the bottom and A_s' is to be placed at the top.

3.4.1.2.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel required in a beam section is given by the minimum of the two following limits:

$$A_s \ge \max\left\{\frac{3\sqrt{f'_c}}{f_y}\ b_w d \text{ and } \frac{200}{f_y} b_w d \quad \text{or}\right. \qquad \text{(ACI 10.5.1)}$$

$$A_s \ge \frac{4}{3}\ A_{s(required)} \qquad \text{(ACI 10.5.3)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{T - beam} \end{cases}$$

$$A_s' \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{T - beam} \end{cases}$$

3.4.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at two stations at the end of each beam element. In designing the shear reinforcement for a particular beam for a particular loading combination at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_u
- Determine the shear force, V_c, that can be resisted by the concrete
- Determine the reinforcement steel required to carry the balance

The following three sections describe in detail the algorithms associated with the preceding steps.

3.4.2.1 Determine Shear Force

In the design of the beam shear reinforcement of a concrete beam, the shear forces for a particular load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the corresponding load combination factors.

3.4.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, Vc, is calculated as follows:

$$V_c = 2\sqrt{f_c'}\, b_w d\,. \qquad \text{(ACI 11.3.1.1)}$$

A limit is imposed on the value of $\sqrt{f_c'}$ as $\sqrt{f_c'} \leq 100$. (ACI 11.1.2)

3.4.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum limit of

$$V_{max} = V_c + (8\sqrt{f_c'})\, b_w d. \qquad \text{(ACI 11.5.6.9)}$$

Given V_u, V_c and V_{max}, the required shear reinforcement is calculated as follows where, φ, the strength reduction factor, is 0.75 (ACI 9.3.2.3).

- If $V_u \leq (V_c / 2)\ \varphi$,

$$\frac{A_v}{s} = 0, \qquad \text{(ACI 11.5.5.1)}$$

else if $(V_c / 2)\ \varphi < V_u \leq \varphi V_{max}$,

$$\frac{A_v}{s} = \frac{(V_u - \varphi V_c)}{\varphi f_{ys} d}, \qquad \text{(ACI 11.5.6.2)}$$

$$\frac{A_v}{s} \geq \max\left(\frac{0.75\sqrt{f_c'}}{f_y} b_w,\ \ \frac{50}{f_y} b_w\right), \qquad \text{(ACI 11.5.5.3)}$$

else if $V_u > \varphi V_{max}$,

a failure condition is declared. (ACI 11.5.6.9)

The maximum of all the calculated A_v/s values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

The beam shear reinforcement requirements displayed by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

3.5 Slab Design

As is done in conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (ACI 318-02) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

3.5.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Design flexural reinforcement for the strip

These two steps, described in the following sections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

3.5.1.1 Determine Factored Moments for the Strip

For each element within the design strip, the program calculates the nodal reactive moments for each load combination. The nodal moments are then added to get the strip moments.

3.5.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip given the bending moment is identical to the design of rectangular beam sections described earlier (or to the T-beam if the slab is ribbed). Where the slab properties (depth, etc.) vary over the width of the strip, the program automatically designs slab widths of each property separately for the bending moment to which they are subjected before summing up the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

3.5.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limits (ACI 7.12.2):

$$A_s \geq 0.0018 \text{ bh } \frac{60000}{f_y}, \qquad \text{(ACI 7.12.2.1)}$$

$$0.0014\ bh \leq A_s \leq 0.0020\ bh. \qquad \text{(ACI 7.12.2.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

3.5.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the following sections.

3.5.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of *d/2* from the face of the support (ACI 11.12.1.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (ACI 11.12.1.3).

3.5.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$,

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \text{ and} \qquad \text{(ACI 13.5.3.2)}$$

$$\gamma_v = 1 - \gamma_f, \qquad \text{(ACI 13.5.3.1)}$$

where b_1 is the width of the critical section measured in the direction of the span and b_2 is the width of the critical section measured in the direction perpendicular to the span.

3.5.2.3 Determination of Concrete Capacity

The concrete punching shear stress capacity is taken as the minimum of the following three limits:

$$v_c = min\begin{cases} \varphi\left(2+\dfrac{4}{\beta_c}\right)\sqrt{f_c'} \\ \varphi\left(2+\dfrac{a_s d}{b_o}\right)\sqrt{f_c'} \\ \varphi 4\sqrt{f_c'} \end{cases} \qquad \text{(ACI 11.12.2.1)}$$

where, β_c is the ratio of the minimum to the maximum dimensions of the critical section, b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 40 & \text{for interior columns,} \\ 30 & \text{for edge columns, and} \\ 20 & \text{for corner columns.} \end{cases} \qquad \text{(ACI 11.12.2.1)}$$

A limit is imposed on the value of $\sqrt{f_c'}$ as $\sqrt{f_c'} \le 100$.(ACI 11.1.2)

3.5.2.4 Determination of Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by SAFE.

Chapter 4
Design for BS 8110-85

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the British limit state design code BS 8110 (BSI 1989). Various notations used in this chapter are listed in Table 4-1. For referencing to the pertinent sections of the British code in this chapter, a prefix "BS" followed by the section number is used.

The design is based on user-specified loading combinations. However, the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-Millimeter-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-Millimeter-Second units unless otherwise noted.

4.1 Notations

Table 4-1 List of Symbols Used in the BS 8110-85 Code

A_{cv}	Area of section for shear resistance, mm^2
A_g	Gross area of cross-section, mm^2

Table 4-1 List of Symbols Used in the BS 8110-85 Code

A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sv}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sv} / s_v	Area of shear reinforcement per unit length of the member, mm^2/mm
a	Depth of compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d	Effective depth of tension reinforcement, mm
d'	Depth to center of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_{cu}	Characteristic cube strength at 28 days, MPa
$f_s^{'}$	Compressive stress in beam compression steel, MPa
f_y	Characteristic strength reinforcement, MPa
f_{yv}	Characteristic strength of link reinforcement, MPa (<460 MPA)
h	Overall depth of a section in the plane of bending, mm
h_f	Flange thickness, mm
K	Normalized design moment, M/bd^2f_{cu}
K'	Limiting normalized moment for a singly reinforced concrete section, taken as 0.156
k_1	Shear strength enhancement factor for support compression

Table 4-1 List of Symbols Used in the BS 8110-85 Code

k_2	Concrete shear strength factor, $\left[f_{cu}/25\right]^{1/3}$
M	Design moment at a section, MPa
M_{single}	Limiting moment capacity as a singly reinforced beam, MPa
s_v	Spacing of the links along the length of the beam, in
T	Tension force, N
V	Design shear force at ultimate design load, N
u	Perimeter of the punch critical section, mm
v	Design shear stress at a beam cross-section or at a punch critical section, MPa
v_c	Design ultimate shear stress resistance of a concrete beam, MPa
$v_{\max}$	Maximum permitted design factored shear stress at a beam section or at the punch critical section, MPa
x	Neutral axis depth, mm
x_{bal}	Depth of neutral axis in a balanced section, mm
z	Lever arm, mm
β_b	Moment redistribution factor in a member
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material strength
ε_c	Maximum concrete strain, 0.0035
ε_s	Strain in tension steel
ε_s'	Strain in compression steel

4.2 Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (DL), live load (LL), pattern live load (PLL), wind (WL), and earthquake (EL) loads, and considering that wind and earthquake forces are reversible, then the following load combinations must be considered (BS 2.4.3):

1.4 DL
1.4 DL + 1.6 LL (BS 2.4.3.1.1)

1.4 DL + 1.6 PLL

1.0 DL ±1.4 WL
1.4 DL ± 1.4 WL (BS 2.4.3.1.1)
1.2 DL + 1.2 LL ± 1.2 WL

1.0 DL ± 1.4 EL
1.4 DL ± 1.4 EL
1.2 DL + 1.2 LL ±1.2 EL

These are also the default design load combinations in SAFE whenever the BS 8110-85 code is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

4.3 Design Strength

The design strength for concrete and steel are obtained by dividing the characteristic strength of the material by a partial factor of safety, γ_m. The values of γ_m used in the program are listed below, which are taken from BS Table 2.2 (BS 2.4.4.1):

Values of γ_m for the ultimate limit state	
Reinforcement	1.15
Concrete in flexure and axial load	1.50
Shear strength without shear reinforcement	1.25

These factors are already incorporated in the design equations and tables in the code. SAFE does not allow them to be overwritten.

4.4 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based upon the beam moments, shear forces, load combination factors, and other criteria described below. The reinforcement requirements are calculated at two check stations at the ends of the beam elements.

All the beams are designed for major direction flexure and shear only. Effects due to any axial forces, minor direction bending, and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design beam flexural reinforcement
- Design beam shear reinforcement

4.4.1 Design Beam Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the end of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine the maximum factored moments
- Determine the reinforcing steel

4.4.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations at that section. Positive beam moments produce bottom steel. In such cases, the beam may be designed as a Rectangular or a T-beam. Negative beam moments produce top steel. In such cases, the beam is always designed as a rectangular section.

4.4.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 4-1. Furthermore, it is assumed that moment redistribution in the member does not exceed 10% (i.e., $\beta_b \geq 0.9$; BS 3.4.4.4). The code also places a limitation on the neutral axis depth, $x/d \leq 0.5$, to safeguard against non-ductile failures (BS 3.4.4.4). In addition, the area of compression reinforcement is calculated on the assumption that the neutral axis depth remains at the maximum permitted value.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams) is summarized in the following sections. It is assumed that the design ultimate axial force does not exceed $0.1\, f_{cu} A_g$ (BS 3.4.4.1); hence all of the beams are designed for major direction flexure and shear only.

4.4.1.2.1 Design of a Rectangular Beam

For rectangular beams, the limiting moment capacity as a singly reinforced beam, M_{single}, is obtained first for a section. The reinforcing steel area is determined based on whether M is greater than, less than, or equal to M_{single}. See Figure 4-1.

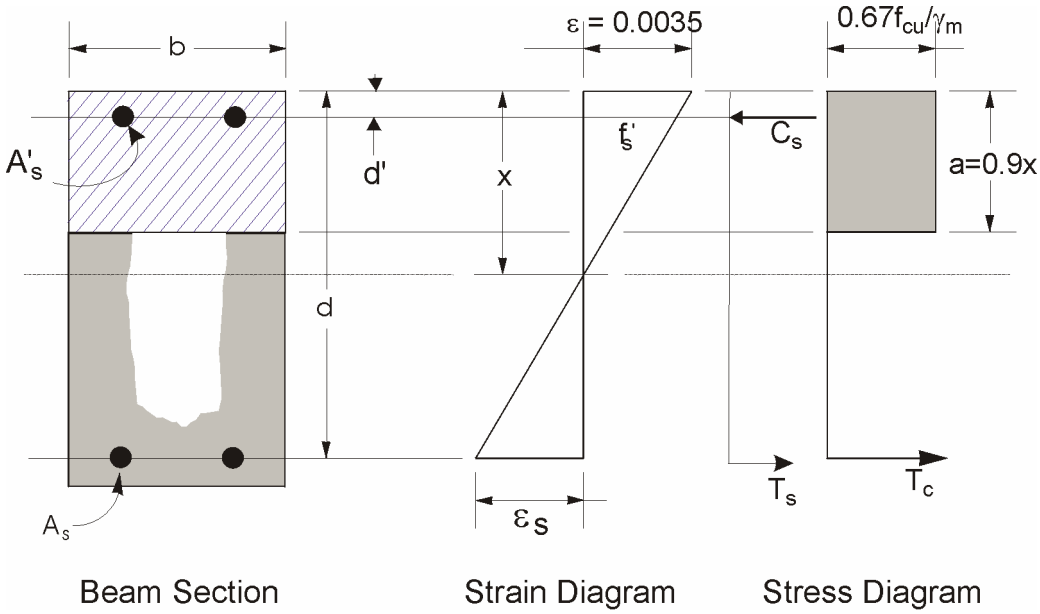


Figure 4-1 Design of Rectangular Beam Section

- Calculate the ultimate limiting moment of resistance of the section as singly reinforced.

  $M_{\text{single}} = K' f_{cu} b d^2$, where (BS 3.4.4.4)

  $K' = 0.156$.

- If $M \le M_{single}$, the area of tension reinforcement, A_s, is obtained from

  $$A_s = \frac{M}{(0.87 f_y) z}, \text{ where} \quad \text{(BS 3.4.4.4)}$$

  $$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d, \text{ and} \quad \text{(BS 3.4.4.4)}$$

  $$K = \frac{M}{f_{cu} b d^2}. \quad \text{(BS 3.4.4.4)}$$

  This is the top steel if the section is under negative moment and the bottom steel if the section is under positive moment.

- If $M > M_{single}$, the area of compression reinforcement, A_s', is given by

$$A_s' = \frac{M - M_{\text{single}}}{f_s'\left(d - d'\right)}, \qquad \text{(BS 3.4.4.4)}$$

where d' is the depth of the compression steel from the concrete compression face, and

$$f_s' = E_s \varepsilon_c \left[1 - \frac{d'}{x_{max}}\right] \le 0.87 f_y. \qquad \text{(BS 3.4.4.4, 2.5.3)}$$

This is the bottom steel if the section is under negative moment. From equilibrium, the area of tension reinforcement is calculated as

$$A_s = \frac{M_{single}}{\left(0.87\ f_y\right)z} + \frac{M - M_{single}}{\left(0.87\ f_y\right)\left(d - d'\right)}, \text{ where} \qquad \text{(BS 3.4.4.4)}$$

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K'}{0.9}}\right\} = 0.777d, \qquad \text{(BS 3.4.4.4)}$$

$$x_{max} = \left(d - z\right)/0.45. \qquad \text{(BS 3.4.4.4)}$$

4.4.1.2.2 Design as a T-Beam

(i) Flanged Beam Under Negative Moment

The contribution of the flange to the strength of the beam is ignored. The design procedure is therefore identical to the one used for rectangular beams, except that in the corresponding equations *b* is replaced by b_w.

(ii) Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially, the neutral axis is assumed to be located in the flange. On the basis of this assumption, the program calculates the exact depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width b_f. If the stress block extends beyond the flange width, the contribution of the web to the flexural strength of the beam is taken into account. See Figure 4-2.

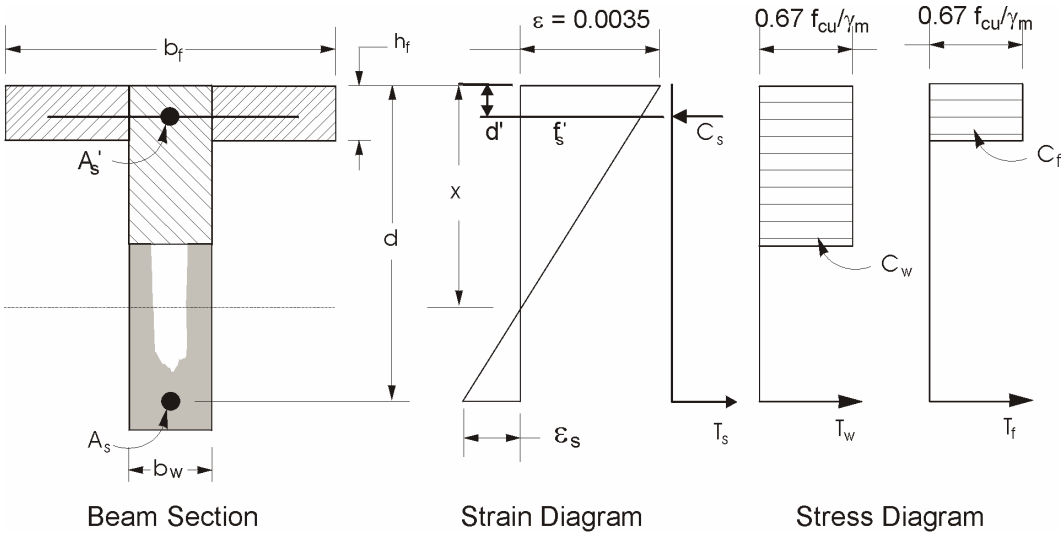


Figure 4-2 Design of a T-Beam Section

Assuming the neutral axis to lie in the flange, the normalized moment is given by

$$\mathrm{K} = \frac{M}{f_{cu}\, b_f\, d^2}. \qquad \text{(BS 3.4.4.4)}$$

Then the moment arm is computed as

$$\mathrm{z} = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d, \qquad \text{(BS 3.4.4.4)}$$

the depth of neutral axis is computed as

$$x = \frac{1}{0.45}(d - z), \text{ and} \qquad \text{(BS 3.4.4.4)}$$

the depth of compression block is given by

$$a = 0.9x. \qquad \text{(BS 3.4.4.4)}$$

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular section design. However, in this case,

the width of the beam is taken as b_f. Whether compression reinforcement is required depends on if $K > K'$.

– If $a > h_f$, calculation for A_s is performed in two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 4-2.

In this case, the ultimate resistance moment of the flange is given by

$$M_f = 0.45 f_{cu} (b_f - b_w) \min(h_f, a_{max}) [d - 0.5\min(h_f, a_{max})], \qquad \text{(BS 3.4.4.5)}$$

the moment taken by the web is computed as

$M_w = M - M_f$, and

the normalized moment resisted by the web is given by

$$K_w = \frac{Mw}{f_{cu} b_w d^2}. \qquad \text{(BS 3.4.4.4)}$$

In this case, the ultimate resistance moment of the flange is given by

$$M_f = 0.45 f_{cu} (b_f - b_w) \min(h_f, a_{max}) [d - 0.5\min(h_f, a_{max})], \qquad \text{(BS 3.4.4.5)}$$

the moment taken by the web is computed as

$M_w = M - M_f$, and

the normalized moment resisted by the web is given by

$$K_w = \frac{Mw}{f_{cu} b_w d^2}. \qquad \text{(BS 3.4.4.4)}$$

– If $K_w \le K'$ (BS 3.4.4.4), the beam is designed as a singly reinforced concrete beam. The area of steel is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.

$$A_s = A_s = \frac{M_f}{0.87 f_y \left[d - 0.5 min\left(h_f, a_{\max}\right) \right]} + \frac{M_w}{0.87 f_y z}, \text{ where}$$

$$z = d \left\{ 0.5 + \sqrt{0.25 - \frac{K_w}{0.9}} \right\} \le 0.95d.$$

– If $K_w > K'$ (BS 3.4.4.4), compression reinforcement is required and is calculated as follows:

The ultimate moment of resistance of the web only is given by

$$M_{uw} = K' f_{cu} b_w d^2. \qquad \text{(BS 3.4.4.4)}$$

The compression reinforcement is required to resist a moment of magnitude $M_w - M_{uw}$. The compression reinforcement is computed as

$$A_s' = \frac{M_w - M_{uw}}{f_s'(d - d')},$$

where, d' is the depth of the compression steel from the concrete compression face, and

$$f_s' = E_s \varepsilon_c \left[1 - \frac{d'}{x_{max}}\right] \le 0.87 f_y. \qquad \text{(BS 3.4.4.4, 2.5.3)}$$

The area of tension reinforcement is obtained from equilibrium

$$A_s = \frac{M_f}{0.87 f_y (d - 0.5 h_f)} + \frac{M_{uw}}{0.87 f_y (0.777 d)} + \frac{M_w - M_{uw}}{0.87 f_y (d - d')}.$$

4.4.1.2.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel required for a beam section is given by the following table, which is taken from BS Table 3.27 (BS 3.12.5.3) with interpolation for reinforcement of intermediate strength:

Section	Situation	Definition of Percentage	Minimum Percentage	
			f_y = 250 MPa	f_y = 460 MPa
Rectangular	—	$100 \frac{A_s}{bh}$	0.24	0.13
T-Beam with web in tension	$\frac{b_w}{b_f} < 0.4$	$100 \frac{A_s}{b_w h}$	0.32	0.18
	$\frac{b_w}{b_f} \ge 0.4$	$100 \frac{A_s}{b_w h}$	0.24	0.13
T-Beam with web in compression	—	$100 \frac{A_s}{b_w h}$	0.48	0.26

The minimum flexural compression steel, if it is required at all, provided in a rectangular or T-beam section is given by the following table, which is taken from BS Table 3.27 (BS 3.12.5.3) with interpolation for reinforcement of intermediate strength:

Section	Situation	Definition of Percentage	Minimum Percentage
Rectangular	—	$100\ \frac{A_s'}{bh}$	0.20
T-Beam	Web in tension	$100\ \frac{A_s'}{b_f h_f}$	0.40
	Web in compression	$100\ \frac{A_s'}{b_w h}$	0.20

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (BS 3.12.6.1).

4.4.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each loading combination in the major direction of the beam. In designing the shear reinforcement for a particular beam for a particular loading combination, the following steps are involved (BS 3.4.5):

- Calculate the design shear stress as

$$v = \frac{V}{A_{cv}},\ A_{cv} = b_w d, \text{ where} \qquad \text{(BS 3.4.5.2)}$$

$$v \le v_{max}, \text{ and} \qquad \text{(BS 3.4.5.2)}$$

$$v_{max} = \min\left(0.8\ \sqrt{f_{cu}},\ 5\ \text{MPa}\right). \qquad \text{(BS 3.4.5.2)}$$

- Calculate the design concrete shear stress from

$$v_c = \frac{0.79\ k_1 k_2}{\gamma_m}\left(\frac{100\ A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}, \qquad \text{(BS 3.4.5.4)}$$

where,

k_1 is the enhancement factor for support compression, and is conservatively taken as 1, (BS 3.4.5.8)

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3} \geq 1, \text{ and} \quad \text{(BS 3.4.5.4)}$$

$$\gamma_m = 1.25. \quad \text{(BS 3.4.5.2)}$$

However, the following limitations also apply:

$$0.15 \leq \frac{100\,A_s}{bd} \leq 3, \quad \text{(BS 3.4.5.4)}$$

$$\frac{400}{d} \geq 1, \text{ and} \quad \text{(BS 3.4.5.4)}$$

$f_{cu} \leq 40$ MPa (for calculation purpose only). (BS 3.4.5.4)

A_s is the area of tensile steel.

- Given v, v_c and v_{max}, the required shear reinforcement in area/unit length is calculated as follows (BS Table 3.8, BS 3.4.5.3):

If $v \leq v_c + 0.4$,

$$\frac{A_{sv}}{s_s} = \frac{0.4\,b_w}{0.87\,f_{yv}}, \quad \text{(BS 3.4.5.3)}$$

else if $(v_c + 0.4) < v \leq v_{max}$,

$$\frac{A_{sv}}{s_s} = \frac{(v - v_c)\,b_w}{0.87\,f_{yv}}. \quad \text{(BS 3.4.5.3)}$$

else if $v > v_{max}$,

a failure condition is declared. (BS 3.4.5.2)

In the preceding expressions, a limit is imposed on the f_{yv} as

$f_{yv} \geq 460$ MPa. (BS 3.4.5.1)

The maximum of all the calculated A_{sv}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

The beam shear reinforcement requirements displayed by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

4.5 Slab Design

As is done in conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method for reinforced concrete (BS 8110-85) as described in the following sections. To learn more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

4.5.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Design flexural reinforcement for the strip

These two steps, described in the following sections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

4.5.1.1 Determine Factored Moments for the Strip

For each element within the design strip, the program calculates the nodal reactive moments for each load combination. The nodal moments are then added to get the strip moments.

4.5.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip given the bending moment is identical to the design of rectangular beam sections described earlier. Where the slab properties (depth, etc.) vary over the width of the strip, the program automatically designs slab widths of each property separately for the bending moment they are subjected to before summing up the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

4.5.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement required in each direction of a slab is given by the following limit (BS 3.12.5.3, BS Table 3.27) with interpolation for reinforcement of intermediate strength:

$$A_s \geq \begin{cases} 0.0024bh & \text{if} \quad f_y \leq 250MPa \\ 0.0013bh & \text{if} \quad f_y \geq 460MPa \end{cases} \qquad \text{(BS 3.12.5.3)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (BS 3.12.6.1).

4.5.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the following sections.

4.5.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of 1.5*d* from the face of the support (BS 3.7.7.4). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (BS 3.7.7.1).

4.5.2.2 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as follows (BS 3.7.7.4):

$$v_c = \frac{0.79\, k_1 k_2}{\gamma_m} \left(\frac{100\, A_s}{bd} \right)^{1/3} \left(\frac{400}{d} \right)^{1/4}, \quad \text{where,} \qquad \text{(BS 3.4.5.4)}$$

k_1 is the enhancement factor for support compression and is conservatively taken as 1, (BS 3.4.5.8)

$$k_2 = \left(\frac{f_{cu}}{25} \right)^{1/3} \geq 1, \text{ and} \qquad \text{(BS 3.4.5.4)}$$

$$\gamma_m = 1.25. \qquad \text{(BS 3.4.5.2)}$$

However, the following limitations also apply:

$$0.15 \leq \frac{100\, A_s}{bd} \leq 3, \qquad \text{(BS 3.4.5.4)}$$

$$\frac{400}{d} \geq 1, \qquad \text{(BS 3.4.5.4)}$$

$v \leq \min (0.8 \sqrt{f_{cu}}$, 5MPa), and (BS 3.4.5.2)

$f_{cu} \leq 40$ MPa (for calculation purpose only). (BS 3.4.5.4)

A_s = area of tensile steel, which is taken as zero in the current implementation.

4.5.2.3 Determination of Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the nominal design shear stress, v, is calculated from the following equation:

$$v = \frac{V_{eff}}{u\,d}, \text{ where} \qquad \text{(BS 3.7.7.3)}$$

$$V_{eff} = V\left\{f + 1.5\frac{M_y}{V\,x} + 1.5\frac{M_x}{V\,y}\right\}, \qquad \text{(BS 3.7.6.2 and BS 3.7.6.3)}$$

u is the perimeter of the critical section,

x and y are the length of the side of the critical section parallel to the axis of bending,

M_x and M_y are the design moment transmitted from the slab to the column at connection,

V is the total punching shear force, and

f is a factor to consider the eccentricity of punching shear force and is taken as

$$f = \begin{cases} 1.00 & \text{for interior columns,} \\ 1.25 & \text{for edge columns, and} \\ 1.25 & \text{for corner columns.} \end{cases} \qquad \text{(BS 3.7.6.2 and BS 3.7.6.3)}$$

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by SAFE.

Chapter 5
Design for CSA A23.3-04

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the Canadian code, CSA A23.3-04 (CSA 04). Various notations used in this chapter are listed in Table 5-1. For referencing to the pertinent sections of the Canadian code in this chapter, a prefix "CSA" followed by the section number is used.

The design is based on user-specified loading combinations. However, the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures. The code is based on Newton-Millimeter-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-Millimeter-Second units unless otherwise noted.

5.1 Notations

Table 5-1 List of Symbols Used in the Canadian Code

A_s	Area of tension reinforcement, sq-mm
A'_s	Area of compression reinforcement, sq-mm
$A_{s(\text{required})}$	Area of steel required for tension reinforcement, sq-mm

Table 5-1 List of Symbols Used in the Canadian Code

A_v	Area of shear reinforcement, sq-mm
A_v / s	Area of shear reinforcement per unit length of the member, sq-mm/mm
a	Depth of compression block, mm
a_b	Depth of compression block at balanced condition, mm
b	Width of member, mm
b_f	Effective width of flange (T-Beam section), mm
b_w	Width of web (T-Beam section), mm
b_0	Perimeter of the punching critical section, mm
b_1	Width of the punching critical section in the direction of bending, mm
b_2	Width of the punching critical section perpendicular to the direction of bending, mm
c	Depth to neutral axis, mm
c_b	Depth to neutral axis at balanced conditions, mm
d	Distance from compression face to tension reinforcement, mm
d'	Concrete cover to center of reinforcing, mm
d_s	Thickness of slab (T-Beam section), mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_c'	Specified compressive strength of concrete, MPa
f_y	Specified yield strength of flexural reinforcement, MPa
f_{yh}	Specified yield strength of shear reinforcement, MPa

Table 5-1 List of Symbols Used in the Canadian Code

h	Overall depth of a section, mm
I_g	Moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement.
M_f	Factored moment at section, N-mm
s	Spacing of the shear reinforcement along the length of the beam, mm
V_c	Shear resisted by concrete, N
V_{max}	Maximum permitted total factored shear force at a section, N
V_f	Factored shear force at a section, N
V_s	Shear force at a section resisted by steel, N
α_1	Ratio of average stress in rectangular stress block to the specified concrete strength
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
ε_s	Strain in reinforcing steel
φ_c	Strength reduction factor for concrete
φ_s	Strength reduction factor for steel
φ_m	Strength reduction factor for member
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear
λ	Shear strength factor

5.2 Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (D), live load (L), pattern live load (PL), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, then the following load combinations may need to be considered (CSA 8.3.2, Table C.1)

1.4 D	(CSA 8.3.2, Table C.1 Case 1)
1.25 D + 1.50 L	(CSA 8.3.2, Table C.1 Case 2)
1.25 D + 1.50 * 0.75 PL	(CSA 13.8.4.2)
1.25 D + 1.40 W	(CSA 8.3.2 Table C.1 Case 4)
0.9 D ± 1.40 W	(CSA 8.3.2 Table C.1 Case 4)
1.25 D ± 1.40 W	(CSA 8.3.2 Table C.1 Case 4)
1.25 D + 0.5 L ±1.40 W	(CSA 8.3.2 Table C.1 Case 4)
1.00 D ±1.00 E	(CSA 8.3.2 Table C.1 Case 5)
1.00 D + (0.50 L ± 1.00 E)	(CSA 8.3.2 Table C.1 Case 5)

These are also the default design load combinations in SAFE whenever the CSA A23.3-04 code is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

5.3 Material Properties

The upper and lower limits of f_c' should be 80 MPa and 20 MPa respectively, for all framing types (CSA 8.6.1.1, 21.2.6.1).

$$f_c' \le 80\,\text{MPa} \qquad \text{(CSA 8.6.1.1, 21.2.6.1)}$$

$f_c' \geq 20\,\text{MPa}$ (CSA 8.6.1.1)

The upper limit of f_y should be 500 MPa for all frames (CSA 8.5.1, 21.2.7.1)

5.4 Strength Reduction Factors

The strength reduction factor, φ, is material dependent and is defined as

φ = 0.65 for concrete and (CSA 8.4.2)

φ = 0.85 for steel. (CSA 8.4.3a)

The user is allowed to overwrite these values. However, caution is advised.

5.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on the beam moments, shear forces, load combination factors, and other criteria described in the sections that follow. The reinforcement requirements are calculated at the end of the beam elements.

All the beams are only designed for major direction flexure and shear. Effects due to any axial forces, minor direction bending, and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design beam flexural reinforcement
- Design beam shear reinforcement

5.5.1 Design Beam Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the end of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine the maximum factored moments
- Determine the reinforcing steel

5.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments produce bottom steel. In such cases the beam may be designed as a Rectangular or a T-beam. Negative beam moments produce top steel. In such cases, the beam is always designed as a rectangular section.

5.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 5-1 (CSA 10.1.7). Furthermore, it is assumed that the compression carried by concrete is less than or equal to that which can be carried at the balanced condition (CSA 10.1.4). When the applied moment exceeds the moment capacity at the balanced condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by SAFE for both rectangular and flanged sections (L- and T-beams) is summarized in the following sections. It is assumed that the design ultimate axial force in a beam is negligible; hence all the beams are designed for major direction flexure and shear only.

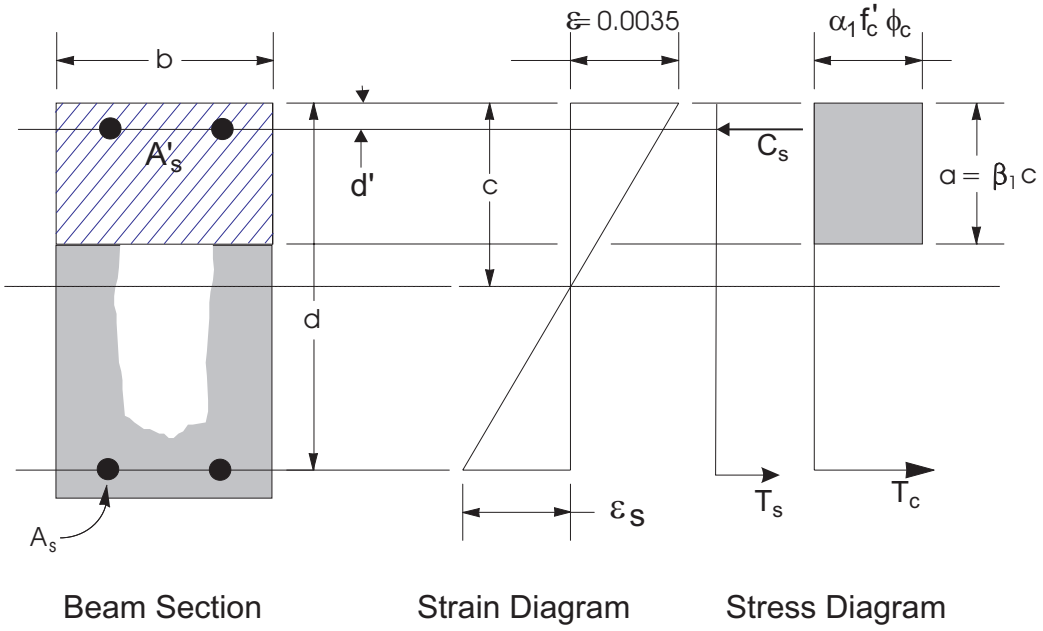


Figure 5-1 Design of a Rectangular Beam Section

5.5.1.2.1 Design for Flexure of a Rectangular Beam

In designing for a factored negative or positive moment, M_f (i.e., designing top or bottom steel), the depth of the compression block is given by a as shown in Figure 5-1, where,

$$a = d - \sqrt{d^2 - \frac{2|M_f|}{\alpha_1 f'_c \varphi_c b}}, \qquad \text{(CSA 10.1)}$$

where the value of φ_c is 0.65 (CSA 8.4.2) in the preceding and following equations. See Figure 5-1. Also α_1, β_1, and c_b are calculated as follows:

$$\alpha_1 = 0.85 - 0.0015\ f'_c \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$\beta_1 = 0.97 - 0.0025\ f'_c \geq 0.67, \text{ and} \qquad \text{(CSA 10.1.7)}$$

$$c_b = \frac{700}{700 + f_y} d. \qquad \text{(CSA 10.5.2)}$$

The balanced depth of compression block is given by

$$a_b = \beta_1 c_b. \qquad \text{(CSA 10.1.7)}$$

- If $a \le a_b$ (CSA 10.5.2), the area of tensile steel reinforcement is then given by

$$A_s = \frac{M_f}{\varphi_s f_y \left(d - \frac{a}{2} \right)}.$$

This steel is to be placed at the bottom if M_f is positive, or at the top if M_f is negative.

- If $a > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

  − The factored compressive force developed in concrete alone is given by

$$C = \varphi_c \alpha_1 f_c' b a_b, \text{ and} \qquad \text{(CSA 10.1.7)}$$

the factored moment resisted by concrete and bottom steel is

$$M_{fc} = C \left(d - \frac{a_b}{2} \right).$$

The moment resisted by compression steel and tensile steel is

$$M_{fs} = M_f - M_{fc}.$$

So the required compression steel is given by

$$A_s' = \frac{M_{fs}}{\left(\varphi_s f_s' - \varphi_c \alpha_1 f_c' \right) \left(d - d' \right)}, \text{ where}$$

$$f_s' = 0.0035 E_s \left[\frac{c - d'}{c} \right] \le f_y. \qquad \text{(CSA 10.1.2 and CSA 10.1.3)}$$

The required tensile steel for the balancing the compression in concrete is

$$A_{s1} = \frac{M_{fc}}{f_y\left(d - \frac{a_b}{s}\right)\varphi_s}, \text{ and}$$

the tensile steel for balancing the compression in steel is

$$A_{s2} = \frac{M_{fc}}{f_y\left(d - d'\right)\varphi_s}.$$

Therefore, the total tensile reinforcement, $A_s = A_{s1} + A_{s2}$, and total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_f is positive, and vice versa.

5.5.1.2.2 Design for Flexure of a T-Beam

(i) Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_f (i.e., designing top steel), the calculation of the steel area is exactly the same as described earlier, i.e., no T-Beam data is to be used.

(ii) Flanged Beam Under Positive Moment

If $M_f > 0$, the depth of the compression block is given by (see Figure 5-2),

$$a = d - \sqrt{d^2 - \frac{2M_f}{\alpha_1 f_c^{'} \varphi_c b_f}}, \qquad \text{(CSA 10.1)}$$

where the value of φ_c is 0.65 (CSA 8.4.2) in the preceding and following equations. See Figure 5-2. Also α_1, β_1, and c_b are calculated as follows:

$$\alpha_1 = 0.85 - 0.0015\ f_c' \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$\beta_1 = 0.97 - 0.0025\ f_c' \geq 0.67\ , \text{and} \qquad \text{(CSA 10.1.7)}$$

$$c_b = \frac{700}{700 + f_y}\ d. \qquad \text{(CSA 10.5.2)}$$

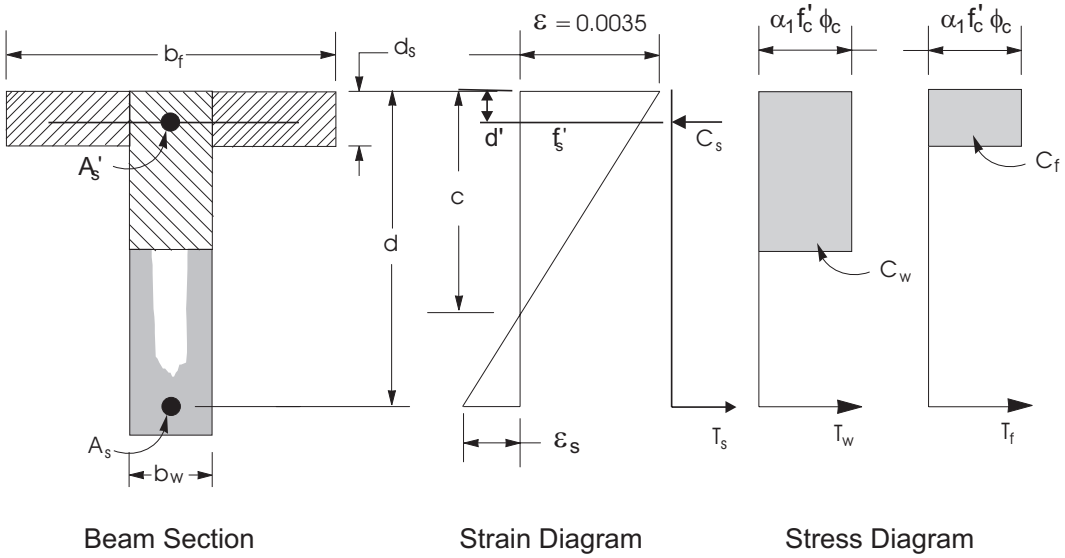


Figure 5-2 Design of a T-Beam Section

The depth of compression block under balanced condition is given by

$$a_b = \beta_1 c_b. \qquad \text{(CSA 10.1.4,10.1.7)}$$

- If $a \le d_s$, the subsequent calculations for A_s are exactly the same as previously done for the rectangular section design. However, in this case, the width of the beam is taken as b_f. Compression reinforcement is required if $a > a_b$.

- If $a > d_s$, calculation for A_s is performed in two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w. As shown in Figure 5-2,

$$C_f = \alpha_1 f_c' (b_f - b_w) \min(d_s, a_{\max}) . \qquad \text{(CSA 10.1.7)}$$

Therefore, $A_{s1} = \dfrac{C_f \varphi_c}{f_y \varphi_s}$ and the portion of M_f that is resisted by the flange is given by

$$M_{ff} = \mathrm{C}_f \left(d - \frac{min\left(d_s, a_{max}\right)}{2} \right) \varphi_c.$$

Therefore, the balance of the moment, M_f, to be carried by the web is given by

$$M_{fw} = M_f - M_{ff}.$$

The web is a rectangular section of dimensions b_w and d, for which the depth of the compression block is recalculated as

$$a_1 = d - \sqrt{d^2 - \frac{2M_{fw}}{\alpha_1 f_c^{'} \varphi_c b_w}}. \qquad \text{(CSA 10.1)}$$

- If $a_1 \le a_b$ (CSA 10.5.2), the area of tensile steel reinforcement is then given by

$$A_{s2} = \frac{M_{fw}}{\varphi_s f_y \left(d - \frac{a_1}{2} \right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2}.$$

This steel is to be placed at the bottom of the T-beam.

- If $a_1 > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

The compressive force in the concrete web alone is given by

$$C = \varphi_c \, \alpha 1 \, f_c' \, b_w a_b, \text{ and} \qquad \text{(CSA 10.1.7)}$$

the moment resisted by the concrete web and tensile steel is

$$M_{fc} = C \left(d - \frac{a_b}{2} \right).$$

The moment resisted by compression steel and tensile steel is

$$M_{fs} = M_{fw} - M_{fc}.$$

Therefore, the compression steel is computed as

$$A_s' = \frac{M_{fs}}{\left(\varphi_s f_c' - \varphi_c \alpha_1 f_c'\right)\left(d - d'\right)}, \text{ where}$$

$$f_s' = 0.0035 E_s \left[\frac{c - d'}{c}\right] \le f_y . \qquad \text{(CSA 10.1.2 and CSA 10.1.3)}$$

The tensile steel for balancing compression in web concrete is

$$A_{s2} = \frac{M_{fc}}{f_y\left(d - \frac{a_b}{2}\right)\varphi_s}, \text{ and}$$

the tensile steel for balancing compression in steel is

$$A_{s3} = \frac{M_{fs}}{f_y\left(d - d'\right)\varphi_s}.$$

Total tensile reinforcement, $A_s = A_{s1} + A_{s2} + A_{s3}$, and total compression reinforcement is A_s'. A_s is to be placed at the bottom and A_s' is to be placed at the top.

5.5.1.2.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel required for a beam section is given by the minimum of the two limits:

$$A_s \ge \frac{0.2\sqrt{f_c'}}{f_y} b_w h, \quad \text{or} \qquad \text{(CSA 10.5.1.2)}$$

$$A_s \ge \frac{4}{3} A_{s(required)}. \qquad \text{(CSA 10.5.1.3)}$$

In addition, the minimum flexural tensile steel provided in a T-section with flange under tension in an ordinary moment resisting frame is given by the limit:

$$A_s \ge 0.004\ (b - b_w)\ d_s. \qquad \text{(CSA 10.5.3.1)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{T - beam} \end{cases}$$

$$A_s' \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{T - beam} \end{cases}$$

5.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at the two stations at the ends of the beam elements. In designing the shear reinforcement for a particular beam for a particular loading combination at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_f
- Determine the shear force, V_c, that can be resisted by the concrete
- Determine the reinforcement steel required to carry the balance

The following three sections describe in detail the algorithms associated with the preceding steps.

5.5.2.1 Determine Shear Force and Moment

In the design of the beam shear reinforcement of a concrete beam, the shear forces and moments for a particular load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the corresponding load combination factors.

5.5.2.2 Determine Concrete Shear Capacity

Given the design force set M_f and V_f, the shear capacity provided by the concrete alone, V_c, is calculated as follows:

$$V_c = \varphi_c \lambda \beta \sqrt{f_c'}\, b_w d, \quad \text{where} \qquad \text{(CSA 11.3.4)}$$

φ_c is the resistance factor for concrete. By default it is taken as 0.65 (CSA8.4.2). For concrete produced in a pre-qualified manufacturing plants, its value can be taken as 0.70 (CSA 16.1.3). This value can be overwritten in the Preferences.

λ is the strength reduction factor to account for low density concrete (CSA 2.2). For normal density concrete, its value is 1 (CSA 8.6.5), which is taken by the program as the default value. For concrete using lower density aggregate, the user can change the value of λ in the material property data. The recommended value for λ is as follows (CSA 8.6.5).

$$\lambda = \begin{cases} 1.00, & \text{for normal density concrete,} \\ 0.85, & \text{for semi-low-density concrete} \\ & \text{in which all of the fine aggregate is natural sand,} \\ 0.75, & \text{for semi-low-density concrete} \\ & \text{in which none of the fine aggregate is natural sand.} \end{cases}$$

β is the factor for accounting for the shear resistance of cracked concrete (CSA 2.2). Its value is normally between 0.1 and 0.4. It is determined in accordance with section 11.3.6 of the Code, which is described in the sections that follow.

b_w is the effective web width. For a rectangular beam, it is the width of the beam. For a T-beam, it is the width of the web of the beam.

d_v is the effective shear depth. It is taken as the greater of 0.9d or 0.72h,

$$d_v = \max\{0.9d, 0.72h\} \qquad \text{(CSA 2.2)}$$

where d is the distance from the extreme compression fiber to the centroid of tension reinforcement, and h is the overall depth of the cross-section in the direction of shear force.

The value of the β factor is preferably taken as the Special value (CSA 11.3.6.2) or it is determined using the Simplified method (CSA 11.3.6.3), if applicable. When the conditions of the Special value or Simplified method

do not apply, the General method is used to determine the β factor (CSA 11.3.6.4).

If the overall beam depth, *h*, is less than 250 mm or if the beam depth of a T-beam below the slab is not greater than one-half of the width of web or 350 mm, β is taken as 0.21 (CSA 11.3.6.2)

When the specified yield strength of the longitudinal reinforcing f_y does not exceed 400 MPa, and the specified concrete strength f_c' does not exceed 60 MPa, β is determined in accordance with the Simplified method, as follows (CSA 11.6.3.3):

- When the section contains at least the minimum transverse reinforcement, β is taken as 0.18 (CSA 11.6.3.3a).

$$\beta = 0.18 \qquad \text{(CSA 11.6.3.a)}$$

- When the section contains no transverse reinforcement, β is determined based on the specified maximum nominal size of coarse aggregate, a_g.

  For maximum size of coarse aggregate not less than 20 mm, β is taken as:

$$\beta = \frac{230}{1000 + d_v} \qquad \text{(CSA 11.3.6.3 b)}$$

  where d_v is the effective shear depth expressed in millimeter, which is described in preceding sections.

  For maximum size of coarse aggregate less than 20 mm, β is taken as:

$$\beta = \frac{230}{1000 + s_{ze}} \qquad \text{(CSA 11.3.6.3 c)}$$

  where, $S_{ze} = \frac{35}{15 + a_g} S_z \geq 0.85 S_z.$ (CSA 11.3.6.3.c)

  In the preceding expression, the crack spacing parameter, S_{ze}, shall be taken as the minimum of d_v and the maximum distance between layers of

distributed longitudinal reinforcement. However, S_{ze} is conservatively taken as equal to d_v.

In summary, for Simplified cases, β can be expressed as follows:

$$\beta = \begin{cases} 0.18, & \text{if minimum transverse reinforcement is provided,} \\ \dfrac{230}{1000 + d_v}, & \text{if no transverse reinforcement provided, and } a_g \geq 20\text{mm,} \\ \dfrac{230}{1000 + S_{ze}}, & \text{if no transverse reinforcement provided, and } a_g < 20\text{mm.} \end{cases}$$

- When the specified yield strength of the longitudinal reinforcing $f_y > 400$ MPa, or the specified concrete strength $f_c' > 60$ MPa, β is determined in accordance with the General method as follows (CSA 11.3.6.1, 11.3.6.4):

$$\beta = \frac{0.40}{(1 + 1500\varepsilon_x)} \bullet \frac{1300}{(1000 + S_{ze})}. \qquad \text{(CSA 11.3.6.4)}$$

In the preceding expression, the equivalent crack spacing parameter, S_{ze}, is taken as equal to 300 mm if minimum transverse reinforcement is provided (CSA 11.3.6.4). Otherwise it is determine as stated in the Simplified method.

$$S_{ze} = \begin{cases} 300 & \text{if minimum transverse reinforcement is provided,} \\ \dfrac{35}{15 + a_g} S_z \geq 0.85 S_z & \text{otherwise.} \end{cases}$$

The longitudinal strain, ε_x, at mid-depth of the cross-section is computed from the following equation:

$$\varepsilon_x = \frac{M_f / d_v + V_f}{s(E_s A_s)} \qquad \text{(CSA 11.3.6.4)}$$

In evaluating the ε_x, the following conditions apply:

V_f and M_f are taken as positive quantities. (CSA 11.3.6.4a)

A_s is taken as the total area of longitudinal reinforcement in the beam section. It is taken as the envelope of reinforcement required for all design load combinations. Actual provided reinforcement might be slightly higher than this quantity. The rebar should be developed to achieve full strength (CSA 11.3.6.3 b).

For sections closer than d_v from the face of the support, ε_x is calculated based on M_f and V_f at a section at a distance d_v from the face of the support (CSA 11.3.6.4 d).

An upper limit on ε_x is imposed as follows:

$$e_x \le 0.003 \qquad \text{(CSA 11.3.6.4 f)}$$

ε_x is positive for tensile action.

- For Ductile moment resisting frames, in the region of potential plastic hinge, the shear reinforcement is designed assuming β is equal to zero:

$$\beta = 0. \qquad \text{(CSA 21.3.4.2)}$$

In both the Simplified and General methods, the shear strength of the section due to concrete, v_c, depends on whether the minimum transverse reinforcement is provided. To check this condition, the program performs the design in two passes. In the first pass, it is assumes that no transverse shear reinforcement is needed. When the program determines that shear reinforcement is need, the program performs the second pass with the assumption that at least minimum shear reinforcement is provided.

5.5.2.3 Determine Required Shear Reinforcement

The minimum possible factored shear resistance, $V_{\text{r,max}}$, is computed as follows:

$$V_{\text{r,max}} = 0.25\varphi_c f_c' b_w d. \qquad \text{(CSA 11.3.3)}$$

The shear reinforcement per unit spacing, A_v/s, is computed as follows:

- If $V_f \le V_c$

$$\frac{A_v}{s} = 0 \qquad \text{(CSA 11.3.5.1)}$$

else if $V_c < V_f \le V_{\text{r,max}}$

$$\frac{A_v}{s} = \frac{\left(V_f - V_c\right)\tan\theta}{\varphi_s f_{yh} d_v} \qquad \text{(CSA 11.3.3, 11.3.5.1)}$$

else if $V_f > V_{\text{r,max}}$

a failure condition is declared. (CSA 11.3.3)

A minimum area of shear reinforcement is provided in the following regions (CSA 11.2.8.1):

(a) in regions of flexural members where the factored shear force V_f exceeds V_c,

(b) in regions of beams and columns with an overall depth greater than 750 mm, and

(c) in regions of beams and columns where the factored torsion T_f exceeds 0.25.

Where the minimum shear reinforcement is required by section CSA 11.2.8.1, or by calculations, the minimum area of shear reinforcement per unit spacing is taken as T_{cr}.

$$\frac{A_v}{s} \ge 0.06\frac{\sqrt{f_c'}}{f_y} b_w \qquad \text{(CSA 11.2.8.2)}$$

In the preceding, the term θ is used. Here θ is the angle of inclination of diagonal compressive stresses to the longitudinal axis of the member. The θ value is normally between 22 to 44 degree. It is determined in accordance with section 11.3.6 in the Code.

Similar to the β factor, which was described previously, the value of θ is preferably taken as the Special value (CSA 11.3.6.2) or it is determined using the Simplified method (CSA 11.3.6.3), whenever applicable. The program uses the General method when conditions for the Simplified method are not satisfied (CSA 11.3.6.4).

- If the overall beam depth, *h*, is less than 250 mm or if the depth of the T-beam below the slab is not greater than one-half of the width of web or 350 mm, θ is taken as 42 degree (CSA 11.3.6.2).
- If the specified yield strength of the longitudinal reinforcing f_y does not exceed 400 MPa, and the specified concrete strength f_c' does not exceed 60 MPa, θ is taken to be 35 degree (CSA 11.3.6.3).

$$\theta = 35^{\circ} \text{ for } P_f \le 0 \text{ or } f_y \le 400 \text{ MPa or } 60 \text{ MPa} \qquad \text{(CSA11.3.6.4)}$$

- If the axial force is tensile, the specified yield strength of the longitudinal reinforcing $f_y > 400$ MPa, and the specified concrete strength $f_c' > 60$ MPa, θ is determined using the General method as follows (CSA 11.3.6.4),

$$\theta = 29 + 7000\varepsilon_x \text{ for } P_f < 0 \text{ or } f_y > 400 \text{ MPa or } 60 \text{ MPa} \text{ (CSA11.3.6.4)}$$

  where ε_x is the longitudinal strain at the mid-depth of the cross-section for the factored load. The calculation procedure has been described in preceding sections.

  The maximum of all of the calculated A_v/s values, obtained from each load combination, is reported for the major directions of the beam along with the controlling shear force and associated load combination number.

  The beam shear reinforcement requirements reported by the program are based purely on the considerations herein. Any other minimum stirrup requirements to satisfy spacing and volumetric requirements must be investigated independently of the program by the user.

5.6 Slab Design

As is done in conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method for reinforced concrete as described in the following sections. To learn more about

the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

5.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Design flexural reinforcement for the strip

These two steps, described in the following sections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

5.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, the program calculates the nodal reactive moments for each load combination. The nodal moments are then added to get the strip moments.

5.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip given the bending moment is identical to the design of rectangular beam sections described earlier. When the slab properties (depth, etc.) vary over the width of the strip, the program automatically designs slab widths of each property separately

for the bending moment to which they are subjected and then sums the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

5.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement provided in each direction of a slab is given by the following limit (CSA 13.10.1):

$$A_s \geq 0.0020\,bh. \qquad \text{(CSA 7.8.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

5.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the following.

5.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of *d/2* from the face of the support (CSA 13.3.3.1 and CSA 13.3.3.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CSA 13.3.3.3).

5.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$, where

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \text{ and} \qquad \text{(CSA 13.10.2)}$$

$$\gamma_v = 1 - \frac{1}{1 + (2/3)\sqrt{b_1/b_2}}, \qquad \text{(CSA 13.3.5.3)}$$

where b_1 is the width of the critical section measured in the direction of the span, and b_2 is the width of the critical section measured in the direction perpendicular to the span.

5.6.2.3 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$v_c' = min\begin{cases} \varphi_c\left(1 + \dfrac{2}{\beta_c}\right)0.19\lambda\sqrt{f_c'} \\ \varphi_c\left(0.19 + \dfrac{\alpha_s d}{b_o}\right)\lambda\sqrt{f_c'} \\ \varphi_c 0.38\lambda\sqrt{f_c'} \end{cases} \qquad \text{(CSA 13.3.4.1)}$$

where, β_c is the ratio of the minimum to the maximum dimensions of the critical section, b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 4 & \text{for interior columns,} \\ 3 & \text{for edge columns, and} \\ 2 & \text{for corner columns.} \end{cases} \qquad \text{(CSA 13.3.4.1)}$$

5.6.2.4 Determination of Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by SAFE.

Chapter 6
Design for CSA A23.3-94

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the Canadian code, CSA A23.3-94 (CSA 1994). Various notations used in this chapter are listed in Table 6-1. For referencing to the pertinent sections of the Canadian code in this chapter, a prefix "CSA" followed by the section number is used.

The design is based on user-specified loading combinations. However, the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-Millimeter-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-Millimeter-Second units unless otherwise noted.

6.1 Notations

Table 6-1 List of Symbols Used in the Canadian Code

A_s	Area of tension reinforcement, sq-mm
A'_s	Area of compression reinforcement, sq-mm

Table 6-1 List of Symbols Used in the Canadian Code

$A_{s(required)}$	Area of steel required for tension reinforcement, sq-mm
A_v	Area of shear reinforcement, sq-mm
A_v / s	Area of shear reinforcement per unit length of the member, sq-mm/mm
a	Depth of compression block, mm
a_b	Depth of compression block at balanced condition, mm
b	Width of member, mm
b_f	Effective width of flange (T-Beam section), mm
b_w	Width of web (T-Beam section), mm
b_0	Perimeter of the punching critical section, mm
b_1	Width of the punching critical section in the direction of bending, mm
b_2	Width of the punching critical section perpendicular to the direction of bending, mm
c	Depth to neutral axis, mm
c_b	Depth to neutral axis at balanced conditions, mm
d	Distance from compression face to tension reinforcement, mm
d'	Concrete cover to center of reinforcing, mm
d_s	Thickness of slab (T-Beam section), mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f'_c	Specified compressive strength of concrete, MPa
f_y	Specified yield strength of flexural reinforcement, MPa

Table 6-1 List of Symbols Used in the Canadian Code

f_{ys}	Specified yield strength of shear reinforcement, MPa
h	Overall depth of a section, mm
M_f	Factored moment at section, N-mm
s	Spacing of the shear reinforcement along the length of the beam, in
V_c	Shear resisted by concrete, N
V_{max}	Maximum permitted total factored shear force at a section, lb
V_f	Factored shear force at a section, N
V_s	Shear force at a section resisted by steel, N
α_1	Ratio of average stress in rectangular stress block to the specified concrete strength
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
ε_s	Strain in reinforcing steel
φ_c	Strength reduction factor for concrete
φ_s	Strength reduction factor for steel
φ_m	Strength reduction factor for member
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear
λ	Shear strength factor

6.2 Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (DL), live load (LL), pattern live load (PLL), wind (WL), and earthquake (EL) loads, and considering that wind and earthquake forces are reversible, then the following load combinations may need to be considered (CSA 8.3):

1.25 DL
1.25 DL + 1.50 LL (CSA 8.3.2)

1.25 DL + 1.50 *0.75 PLL (CSA 13.9.4.3)

1.25 DL ± 1.50 WL
0.85 DL ± 1.50 WL (CSA 8.3.2)
1.25 DL + 0.7 (1.50 LL ± 1.50 WL)

1.00 DL ±1.00 EL
1.00 DL + (0.50 LL ± 1.00 EL) (CSA 8.3.2)

These are also the default design load combinations in SAFE whenever the CSA A23.3-94 code is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

6.3 Strength Reduction Factors

The strength reduction factor, φ, is material dependent and is defined as

$\varphi = 0.60$ for concrete and (CSA 8.4.2)

$\varphi = 0.85$ for steel. (CSA 8.4.3)

The user is allowed to overwrite these values. However, caution is advised.

6.4 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on the beam moments, shear forces, load combination factors, and other criteria described in following sections. The reinforcement requirements are calculated at the end of the beam elements.

All the beams are designed for major direction flexure and shear only. Effects due to any axial forces, minor direction bending, and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design beam flexural reinforcement
- Design beam shear reinforcement

6.4.1 Design Beam Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the end of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine the maximum factored moments
- Determine the reinforcing steel

6.4.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments produce bottom steel. In such cases, the beam may be

designed as a Rectangular or T-beam. Negative beam moments produce top steel. In such cases, the beam is always designed as a rectangular section.

6.4.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 6-1 (CSA 10.1.7). Furthermore, it is assumed that the compression carried by concrete is less than or equal to that which can be carried at the balanced condition (CSA 10.1.4). When the applied moment exceeds the moment capacity at the balanced condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

In designing the beam flexural reinforcement, the following limits are imposed on the steel tensile strength and the concrete compressive strength:

$$f_y \leq 500 \text{ MPa} \qquad \text{(CSA 8.5.1)}$$

$$f_c' \leq 80 \text{ MPa} \qquad \text{(CSA 8.6.1.1)}$$

The design procedure used by SAFE for both rectangular and flanged sections (L- and T-beams) is summarized in the following sections. It is assumed that the design ultimate axial force in a beam is negligible, hence all the beams are designed for major direction flexure and shear only.

6.4.1.2.1 Design for Flexure of a Rectangular Beam

In designing for a factored negative or positive moment, M_f (i.e., designing top or bottom steel), the depth of the compression block is given by a, as shown in Figure 6-1, where,

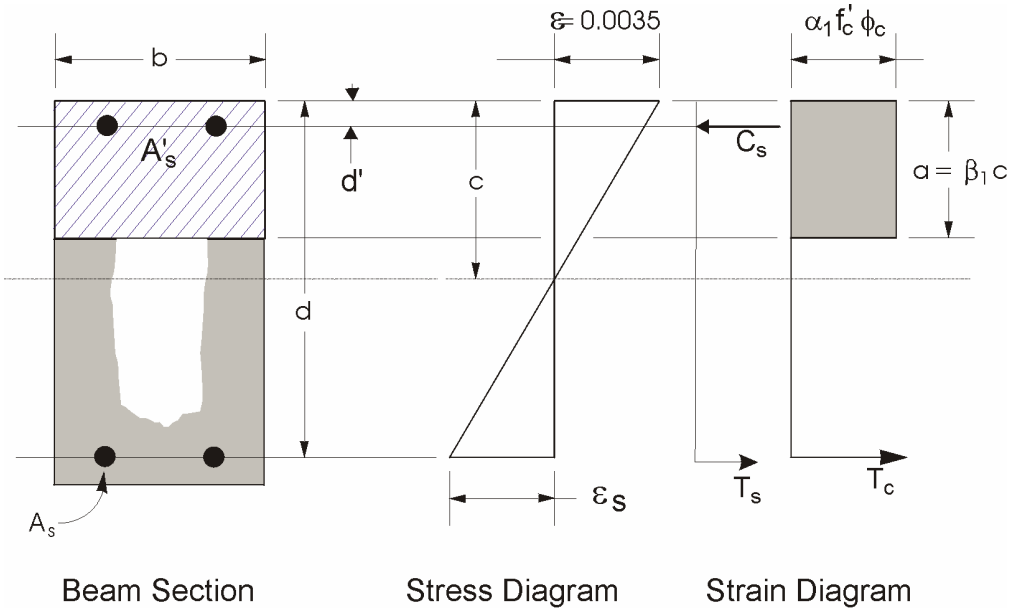


Figure 6-1 Design of a Rectangular Beam Section

$$a = d - \sqrt{d^2 - \frac{2|M_f|}{\alpha_1 f_c' \varphi_c b}}, \qquad \text{(CSA 10.1)}$$

where the value of φ_c is 0.60 (CSA 9.4.2) in the preceding and following equations. See Figure 6-1. Also α_1, β_1, and c_b are calculated as follows:

$$\alpha_1 = 0.85 - 0.0015\ f_c' \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$\beta_1 = 0.97 - 0.0025\ f_c' \geq 0.67, \text{ and} \qquad \text{(CSA 10.1.7)}$$

$$c_b = \frac{700}{700 + f_y} d. \qquad \text{(CSA 10.5.2)}$$

The balanced depth of the compression block is given by

$$a_b = \beta_1 c_b. \qquad \text{(CSA 10.1.7)}$$

- If $a \le a_b$ (CSA 10.5.2), the area of tensile steel reinforcement is then given by

$$A_s = \frac{M_f}{\varphi_s f_y \left(d - \frac{a}{2} \right)}.$$

This steel is to be placed at the bottom if M_f is positive, or at the top if M_f is negative.

- If $a > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

  − The factored compressive force developed in concrete alone is given by

$$C = \varphi_c \alpha_1 f_c' b a_b \text{, and} \qquad \text{(CSA 10.1.7)}$$

the factored moment resisted by concrete and bottom steel is

$$M_{fc} = C \left(d - \frac{a_b}{2} \right).$$

The moment resisted by compression steel and tensile steel is

$$M_{fs} = M_f - M_{fc}.$$

So the required compression steel is given by

$$A_s' = \frac{M_{fs}}{\left(\varphi_s f_s' - \varphi_c \alpha_1 f_c' \right)\left(d - d' \right)} \text{, where}$$

$$f_s' = 0.0035 E_s \left[\frac{c - d'}{c} \right] \le f_y. \qquad \text{(CSA 10.1.2 and CSA 10.1.3)}$$

The required tensile steel for the balancing the compression in concrete is

$$A_{s1} = \frac{M_{fc}}{f_y \left(d - \frac{a_b}{s} \right) \varphi_s} \text{, and}$$

the tensile steel for balancing the compression in steel is

$$A_{s2} = \frac{M_{fc}}{f_y (d - d') \varphi_s}.$$

Therefore, the total tensile reinforcement, $A_s = A_{s1} + A_{s2}$, and total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_f is positive, and vice versa.

6.4.1.2.2 Design for Flexure of a T-Beam

(i) Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_f (i.e., designing top steel), the calculation of the steel area is exactly the same as above, i.e., no T-Beam data is to be used.

(ii) Flanged Beam Under Positive Moment

If $M_f > 0$, the depth of the compression block is given by (see Figure 6-2).

$$a = d - \sqrt{d^2 - \frac{2M_f}{\alpha_1 f_c' \varphi_c b_f}}, \qquad \text{(CSA 10.1)}$$

where the value of φ_c is 0.60 (CSA 9.4.2) in the preceding and following equations. See Figure 6-2. Also α_1, β_1, and c_b are calculated as follows:

$$\alpha_1 = 0.85 - 0.0015 f_c' \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$\beta_1 = 0.97 - 0.0025 f_c' \geq 0.67, \text{ and} \qquad \text{(CSA 10.1.7)}$$

$$c_b = \frac{700}{700 + f_y} d. \qquad \text{(CSA 10.5.2)}$$

The depth of compression block under balanced condition is given by

$$a_b = \beta_1 c_b . \qquad \text{(CSA 10.1.4)}$$

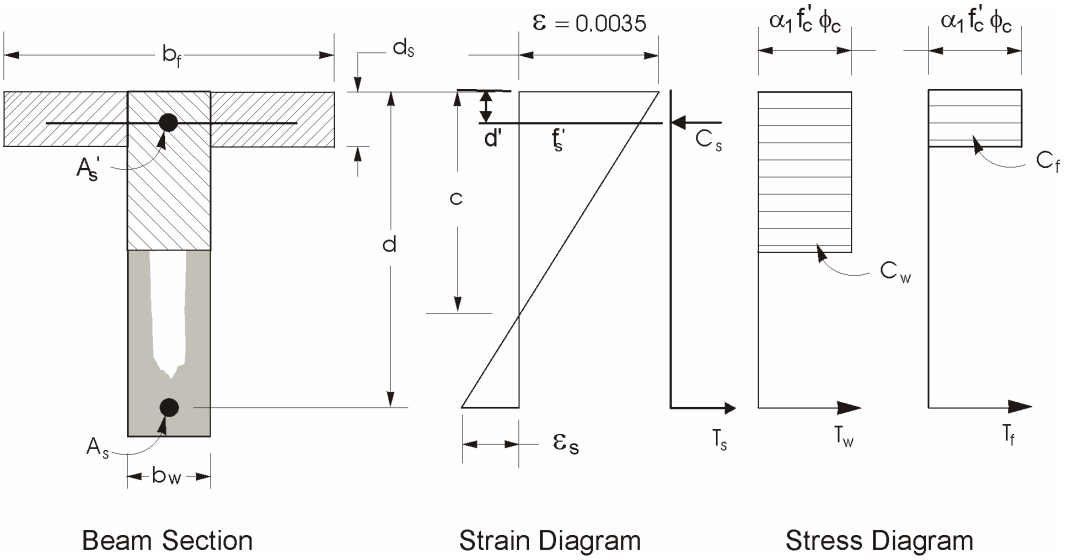


Figure 6-2 Design of a T-Beam Section

- If $a \le d_s$, the subsequent calculations for A_s are exactly the same as previously done for the rectangular section design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required if $a > a_b$.

- If $a > d_s$, calculation for A_s is done in two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w. As shown in Figure 6-2,

$$C_f = \alpha_1 f_c' \ (b_f - b_w) \min(d_s, a_{\max}). \qquad \text{(CSA 10.1.7)}$$

Therefore, $A_{s1} = \dfrac{C_f \varphi_c}{f_y \varphi_s}$ and the portion of M_f that is resisted by the flange is given by

$$M_{ff} = \mathrm{C}_f \left(d - \frac{\min(d_s, a_{max})}{2} \right) \varphi_c.$$

Therefore, the balance of the moment, M_f, to be carried by the web is given by

$$M_{fw} = M_f - M_{ff}.$$

The web is a rectangular section of dimensions b_w and d, for which the depth of the compression block is recalculated as

$$a_1 = d - \sqrt{d^2 - \frac{2M_{fw}}{\alpha_1 f_c' \varphi_c b_w}}. \qquad \text{(CSA 10.1)}$$

- If $a_1 \le a_b$ (CSA 10.5.2), the area of tensile steel reinforcement is then given by

$$A_{s2} = \frac{M_{fw}}{\varphi_s f_y \left(d - \frac{a_1}{2} \right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2}.$$

This steel is to be placed at the bottom of the T-beam.

- If $a_1 > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

The compressive force in the concrete web alone is given by

$$C = \alpha f_c' b_w a_b, \text{ and} \qquad \text{(CSA 10.1.7)}$$

the moment resisted by the concrete web and tensile steel is

$$M_{fc} = C \left(d - \frac{a_b}{2} \right) \varphi_c.$$

The moment resisted by compression steel and tensile steel is

$$M_{fs} = M_{fw} - M_{fc}.$$

Therefore, the compression steel is computed as

$$A_s' = \frac{M_{fs}}{\left(\varphi_s f_c' - \varphi_c \alpha_1 f_c'\right)\left(d - d'\right)}, \text{ where}$$

$$f_s' = 0.0035 E_s \left[\frac{c - d'}{c}\right] \le f_y . \qquad \text{(CSA 10.1.2 and CSA 10.1.3)}$$

The tensile steel for balancing compression in web concrete is

$$A_{s2} = \frac{M_{fc}}{f_y\left(d - \frac{a_b}{2}\right)\varphi_s}, \text{ and}$$

the tensile steel for balancing compression in steel is

$$A_{s3} = \frac{M_{fs}}{f_y\left(d - d'\right)\varphi_s}.$$

Total tensile reinforcement, $A_s = A_{s1} + A_{s2} + A_{s3}$, and total compression reinforcement is A_s'. A_s is to be placed at the bottom and A_s' is to be placed at the top.

6.4.1.2.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel required for a beam section is given by the minimum of the two limits:

$$A_s \ge \frac{0.2\sqrt{f_c'}}{f_y} b_w h, \quad \text{or} \qquad \text{(CSA 10.5.1.2)}$$

$$A_s \ge \frac{4}{3} A_{s(required)}. \qquad \text{(CSA 10.5.1.3)}$$

In addition, the minimum flexural tensile steel provided in a T-section with flange under tension in an ordinary moment resisting frame is given by the limit:

$$A_s \ge 0.004\ (b - b_w)\ d_s. \qquad \text{(CSA 10.5.3.1)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{T - beam} \end{cases}$$

$$A_s' \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{T - beam.} \end{cases}$$

6.4.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at the two stations at the ends of the beam elements. In designing the shear reinforcement for a particular beam for a particular loading combination at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_f.
- Determine the shear force, V_c, that can be resisted by the concrete.
- Determine the reinforcement steel required to carry the balance.

In designing the beam shear reinforcement, the following limits are imposed on the steel tensile strength and the concrete compressive strength:

$$f_{ys} \le 500 \text{ MPa} \qquad \text{(CSA 8.5.1)}$$

$$f_c' \le 80 \text{ MPa} \qquad \text{(CSA 8.6.1.1)}$$

The following three sections describe in detail the algorithms associated with the preceding steps.

6.4.2.1 Determine Shear Force and Moment

In the design of the beam shear reinforcement of a concrete beam, the shear forces and moments for a particular load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the corresponding load combination factors.

6.4.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as follows:

$$V_c = 0.2\varphi_c\lambda\sqrt{f_c'}\ b_w d, \qquad \text{if} \quad d \le 300 \qquad \text{(CSA 11.3.5.1)}$$

$$V_c = \frac{260}{1000 + d}\varphi_c\lambda\sqrt{f_c'}\ b_w d \ge 0.1\ \varphi_c\lambda\sqrt{f_c'}\ b_w d, \quad \text{if} \quad d > 300 \qquad \text{(CSA 11.3.5.2)}$$

where λ is taken as one for normal weight concrete.

6.4.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum limit of

$$V_{max} = V_c + 0.8\ \varphi_c\lambda\sqrt{f_c'}\ \ b_w d. \qquad \text{(CSA 11.3.4)}$$

Given V_u, V_c and V_{max}, the required shear reinforcement in area/unit length is calculated as follows:

- If $V_f \le (V_c / 2)$,

$$\frac{A_v}{s} = 0, \qquad \text{(CSA 11.2.8.1)}$$

else if $(V_c / 2) < V_f \le \left[V_c + \varphi_s\left(0.06\sqrt{f_c'}b_w d\right)\right]$,

$$\frac{A_v}{s} = \frac{0.06\sqrt{f_c'}b_w}{f_{ys}}, \qquad \text{(CSA 11.2.8.4)}$$

else if $\left[V_c + \varphi_s\left(0.06\sqrt{f_c'}b_w d\right)\right] < V_f \le V_{max}$,

$$\frac{A_v}{s} = \frac{\left(V_f - V_c\right)}{\varphi_s f_{ys} d}, \qquad \text{(CSA 11.3.7)}$$

else if $V_f > V_{max}$,

a failure condition is declared. (CSA 11.3.4)

The maximum of all the calculated A_v/s values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

The beam shear reinforcement requirements displayed by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

6.5 Slab Design

As is done in conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

6.5.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to

the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Design flexural reinforcement for the strip

These two steps, described in the following sections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combinations, is obtained and reported.

6.5.1.1 Determine Factored Moments for the Strip

For each element within the design strip, the program calculates the nodal reactive moments for each load combination. The nodal moments are then added to get the strip moments.

6.5.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier. When the slab properties (depth, etc.) vary over the width of the strip, the program automatically designs slab widths of each property separately for the bending moment to which they are subjected and then sums the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

6.5.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement provided in each direction of a slab is given by the following limit (CSA 13.11.1):

$$A_s \geq 0.0020\, bh \qquad \text{(CSA 7.8.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

6.5.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the following sections.

6.5.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of *d/2* from the face of the support (CSA 13.4.3.1 and CSA 13.4.3.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CSA 13.4.3.3).

6.5.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$, where

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \text{ and} \qquad \text{(CSA 13.11.2)}$$

$$\gamma_v = 1 - \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \qquad \text{(CSA 13.4.5.3)}$$

where b_1 is the width of the critical section measured in the direction of the span and b_2 is the width of the critical section measured in the direction perpendicular to the span.

6.5.2.3 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$v_c = min\begin{cases} \varphi_c\left(1+\dfrac{2}{\beta_c}\right)0.2\lambda\sqrt{f_c'} \\ \varphi_c\left(0.2+\dfrac{\alpha_s d}{b_o}\right)\lambda\sqrt{f_c'} \\ \varphi_c 0.4\lambda\sqrt{f_c'} \end{cases} \qquad \text{(CSA 13.4.4)}$$

where, β_c is the ratio of the minimum to the maximum dimensions of the critical section, b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 4 & \text{for interior columns,} \\ 3 & \text{for edge columns, and} \\ 2 & \text{for corner columns.} \end{cases} \qquad \text{(CSA 13.4.4)}$$

Also the following limits are imposed on the steel and concrete strengths:

$$f_y \le 500 \text{ MPa} \qquad \text{(CSA 8.5.1)}$$

$$f_c' \le 80 \text{ MPa} \qquad \text{(CSA 8.6.1.1)}$$

6.5.2.4 Determination of Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by SAFE.

Chapter 7
Design for Eurocode 2

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the European concrete design code, 1993 Eurocode 2 (CEN 1993). Various notations used in this chapter are listed in Table 7-1. For referencing to the pertinent sections of the Eurocode in this chapter, a prefix "EC2" followed by the section number is used.

The design is based on user-specified loading combinations. However, the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-Millimeter-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-Millimeter-Second units unless otherwise noted.

7.1 Notations

Table 7-1 List of Symbols Used in the Eurocode 2

A_c	Area of concrete section, mm^2

Table 7-1 List of Symbols Used in the Eurocode 2

A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sw}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sw}/s_v	Area of shear reinforcement per unit length of the member, mm^2
a	Depth of compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d	Effective depth of tension reinforcement, mm
d'	Effective depth of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_{cd}	Design concrete strength = f_{ck} / γ_c, MPa
f_{ck}	Characteristic compressive concrete cylinder strength at 28 days, MPa
f_{yd}	Design yield strength of reinforcing steel = f_{yk} / γ_s, MPa
f_{yk}	Characteristic strength of shear reinforcement, MPa
f'_s	Compressive stress in beam compression steel, MPa
f_{ywd}	Design strength of shear reinforcement = f_{ywk} / γ_s, MPa
f_{ywk}	Characteristic strength of shear reinforcement, MPa
h	Overall thickness of slab, mm
h_f	Flange thickness, mm

Table 7-1 List of Symbols Used in the Eurocode 2

M	Design moment at a section, N-mm
m	Normalized design moment, $M/bd^2\alpha f_{cd}$
m_{lim}	Limiting normalized moment capacity as a singly reinforced beam
s_v	Spacing of the shear reinforcement along the length of the beam, mm
u	Perimeter of the punch critical section, mm
V_{Rd1}	Design shear resistance from concrete alone, N
V_{Rd2}	Design limiting shear resistance of a cross-section, N
V_{sd}	Shear force at ultimate design load, N
x	Depth of neutral axis, mm
x_{lim}	Limiting depth of neutral axis, mm
α	Concrete strength reduction factor for sustained loading and stress-block
β	Enhancement factor of shear resistance for concentrated load; also the coefficient that takes account of the eccentricity of loading in determining punching shear stress; factor for the depth of compressive stress block
γ_f	Partial safety factor for load
γ_c	Partial safety factor for concrete strength
γ_m	Partial safety factor for material strength
γ_s	Partial safety factor for steel strength
δ	Redistribution factor
ε_c	Concrete strain
ε_s	Strain in tension steel
ν	Effectiveness factor for shear resistance without concrete crushing

Table 7-1 List of Symbols Used in the Eurocode 2

ρ	Tension reinforcement ratio, A_s/bd
ω	Normalized tensile steel ratio, $A_s f_{yd}/\alpha f_{cd} bd$
ω'	Normalized compression steel ratio, $A'_s f_{yd} \gamma_s/\alpha f'_s bd$
ω_{lim}	Normalized limiting tensile steel ratio

7.2 Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (DL), live load (LL), pattern live load (PLL), wind (WL), and earthquake (EL) loads, and considering that wind and earthquake forces are reversible, then the following load combinations must be considered (EC2 2.3.3):

1.35 DL
1.35 DL + 1.50 LL (EC2 2.3.3.1)

1.35 DL + 1.50 PLL

1.35 DL ± 1.50 WL
1.00 DL ± 1.50 WL
1.35 DL + 1.35 LL ± 1.35 WL (EC2 2.3.3.1)

1.00 DL ± 1.00 EL
1.00 DL + 1.5*0.3 LL ± 1.0 EL (EC2 2.3.3.1)

These are also the default design load combinations in SAFE whenever the Eurocode is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

7.3 Design Strength

The design strength for concrete and steel are obtained by dividing the characteristic strength of the material by a partial factor of safety, γ_m. The values of γ_m used in the program are listed in the text that follows. These values are

recommended by the code to give an acceptable level of safety for normal structures under regular design situations (EC2 2.3.3.2). For accidental and earthquake situations, the recommended values are less than the tabulated value. The user should consider those separately.

The partial safety factors for the materials—the design strengths of concrete and steel—are as follows:

- Partial safety factor for steel, $\gamma_s = 1.15$, and (EC2 2.3.3.2)
- Partial safety factor for concrete, $\gamma_c = 1.15$. (EC2 2.3.3.2)

The user is allowed to overwrite these values. However, caution is advised.

7.4 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on the beam moments, shears, load combination factors, and other criteria described in the text that follows. The reinforcement requirements are calculated at two check stations at the ends of the beam elements. All the beams are designed for major direction flexure and shear only. Effects due to any axial forces, minor direction bending, and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design beam flexural reinforcement
- Design beam shear reinforcement

7.4.1 Design Beam Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the ends of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine the maximum factored moments

- Determine the reinforcing steel

7.4.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments produce bottom steel. In such cases, the beam may be designed as a Rectangular or a T-beam. Negative beam moments produce top steel. In such cases, the beam is always designed as a rectangular section.

7.4.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 7-1. The area of the stress block and the depth of the center of the compressive force from the most compressed fiber are taken as,

$$C = \alpha f_{cd} a d \text{ and}$$

$$a = \beta x,$$

where x is the depth of the neutral axis, and α and β are taken respectively as

$$\alpha = 0.8, \quad \text{and} \qquad \text{(EC2 4.2.1.3.3)}$$

$$\beta = 0.8. \qquad \text{(EC2 4.2.1.3.3)}$$

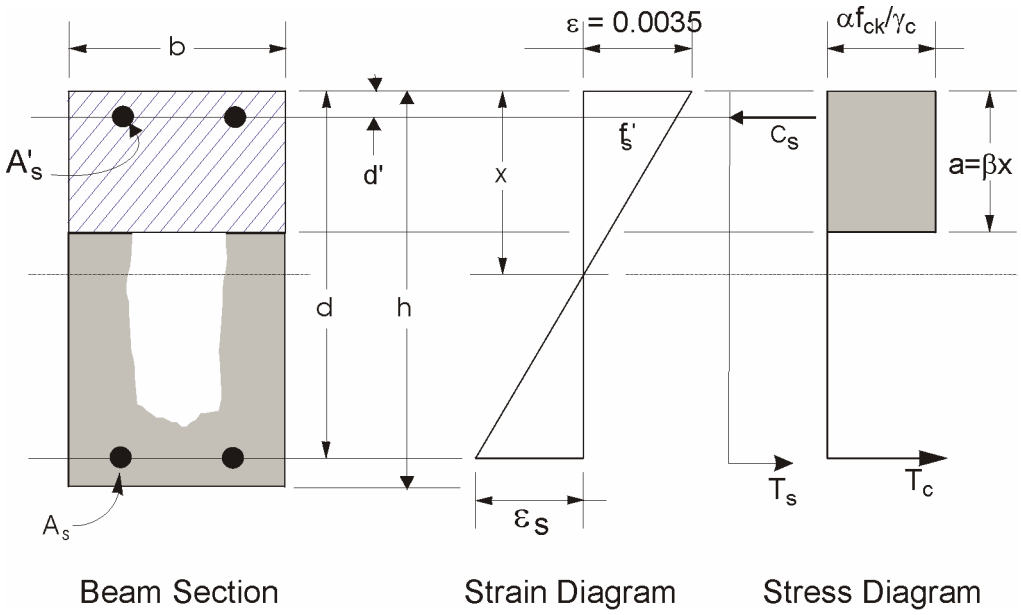


Figure 7-1 Design of a Rectangular Beam Section

α is the reduction factor to account for sustained compression and rectangular stress block. α is generally assumed to be 0.80 for the assumed rectangular stress block (EC2 4.2.1.3.3). The β factor considers the depth of the stress block and it is assumed to be 0.8 (EC2 4.2.1.3.3).

Furthermore, it is assumed that moment redistribution in the member does not exceed the code specified limiting value. The code also places a limitation on the neutral axis depth, to safeguard against non-ductile failures (EC2 2.5.3.4.2). When the applied moment exceeds the limiting moment capacity as a singly reinforced beam, the area of compression reinforcement is calculated on the assumption that the neutral axis depth remains at the maximum permitted value.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams) is summarized in the text that follows. It is assumed that the design ultimate axial force does not exceed $0.08 f_{ck} A_g$ (EC2 4.3.1.2); hence all of the beams are designed for major direction flexure and shear only.

7.4.1.2.1 Design as a Rectangular Beam

For rectangular beams, the normalized moment, m, and the normalized section capacity as a singly reinforce beam, m_{lim}, are obtained first. The reinforcing steel area is determined based on whether m is greater than, less than, or equal to m_{lim}.

- Calculate the normalized design moment, m

$$m = \frac{M}{bd^2 \alpha f_{cd}}, \text{ where}$$

α is the reduction factor to account for sustained compression and the β factor considers the depth of the neutral axis. α is generally assumed to be 0.80 for assumed rectangular stress block (EC2 4.2.1.3.3). α also is generally assumed to be 0.80 for an assumed rectangular stress block, (EC2 4.2.1.3.3). The concrete compression stress block is assumed to be rectangular (see Figure 7-1), with a stress value of αf_{cd}, where f_{cd}, is the design concrete strength and is equal to f_{ck}/γ_c.

- Calculate the normalized concrete moment capacity as a singly reinforced beam, m_{lim}.

$$m_{lim} = \beta\left(\frac{x}{d}\right)_{lim}\left[1 - \frac{\beta}{2}\left(\frac{x}{d}\right)_{lim}\right],$$

where the limiting value of the ratio of the neutral axis depth at the ultimate limit state to the effective depth, $\left[x/d\right]_{lim}$, is expressed as a function of the ratio of the redistributed moment to the moment before redistribution, δ, as follows:

$$\left(\frac{x}{d}\right)_{lim} = \frac{\delta - 0.44}{1.25} \qquad \text{if} \quad f_{ck} \le 35, \qquad \text{(EC2 2.5.3.4.2)}$$

$$\left(\frac{x}{d}\right)_{lim} = \frac{\delta - 0.56}{1.25} \qquad \text{if} \quad f_{ck} > 35, \text{ and} \quad \text{(EC2 2.5.3.4.2)}$$

δ is assumed to be 1.

- If $m \le m_{lim}$, a singly reinforced beam will suffice. Calculate the normalized steel ratio,

  $$\omega = 1 - \sqrt{1-2m}.$$

  Calculate the area of tension reinforcement, A_s, from

  $$A_s = \omega \left[\frac{\alpha f_{cd} bd}{f_{yd}} \right].$$

  This is the top steel if the section is under negative moment and the bottom steel if the section is under positive moment.

- If $m > m_{lim}$,, the beam will not suffice as a singly reinforced beam. Both top and bottom steel are required.

  Calculate the normalized steel ratios ω', ω_{lim}, and ω.

  $$\omega_{lim} = \beta\left(\frac{x}{d}\right)_{\lim} = 1 - \sqrt{1-2m_{\lim}}$$

  $$\omega' = \frac{m - m_{lim}}{1 - d'/d}, \text{ and}$$

  $$\omega = \omega_{lim} + \omega'$$

  where, d' is the depth of the compression steel from the concrete compression face.

  Calculate the area of compression and tension reinforcement, A_s' and A_s, as follows:

  $$A_s' = \omega' \left[\frac{\alpha f_{cd} bd}{f_s'} \right], \text{ and}$$

  $$A_s = \omega \left[\frac{\alpha f_{cd} bd}{f_{yd}} \right],$$

where, f_s' is the stress in the compression steel, and is given by

$$f_s' = E_s \varepsilon_c \left[1 - \frac{d'}{x_{lim}}\right] \leq f_{yd} \qquad \text{(EC2 4.2.2.3.2)}$$

7.4.1.2.2 Design as a T-Beam

(i) Flanged Beam Under Negative Moment

The contribution of the flange to the strength of the beam is ignored if the flange is in the tension side. See Figure 7-2. The design procedure is therefore identical to the one used for rectangular beams. However, the width of the web, b_w, is taken as the width of the beam.

(ii) Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially the neutral axis is assumed to be located within the flange. Based on this assumption, the program calculates the depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width b_f. If the stress block extends beyond the flange, additional calculation is required. See Figure 7-2.

- Calculate the normalized design moment, *m*.

  $m = \dfrac{M}{b_f d^2 \alpha f_{cd}}$, where

  α is the reduction factor to account for sustained compression. α is generally assumed to be 0.80 for the assumed rectangular stress block (EC2 4.2.1.3). See also page 5-7 for α. The concrete compression stress block is assumed to be rectangular, with a stress value of αf_{cd}.

  Calculate the limiting value of the ratio of the neutral axis depth at the ultimate limit state to the effective depth, $[x/d]_{lim}$, which is expressed as a function of the ratio of the redistributed moment to the moment before redistribution, δ, as follows:

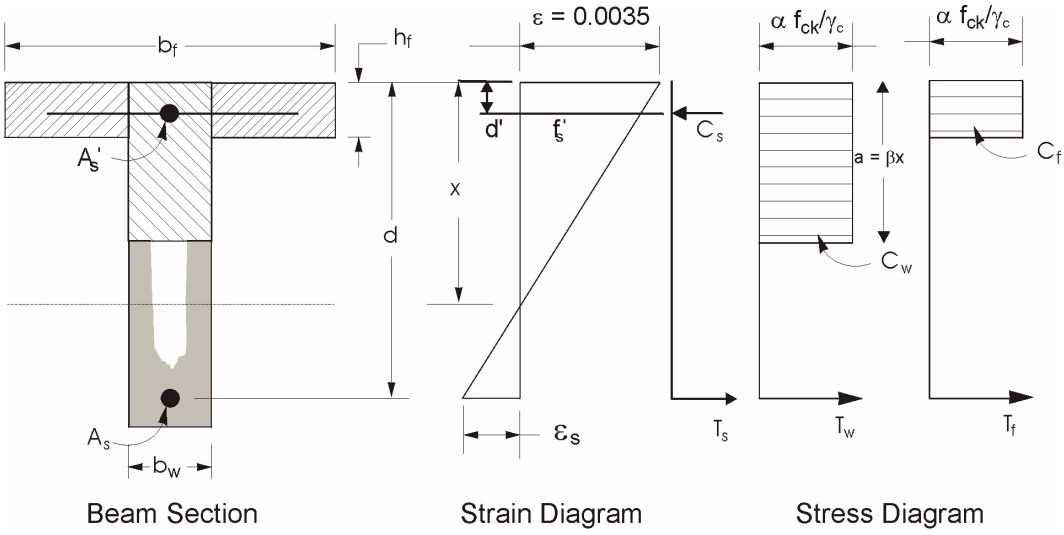


Figure 7-2 Design of a T-Beam Section

$$\left(\frac{x}{d}\right)_{lim} = \frac{\delta - 0.44}{1.25}, \quad \text{if} \quad f_{ck} \le 35, \qquad \text{(EC2 2.5.3.4.1)}$$

$$\left(\frac{x}{d}\right)_{lim} = \frac{\delta - 0.56}{1.25}, \quad \text{if} \quad f_{ck} > 35, \qquad \text{(EC2 2.5.3.4.1)}$$

δ is assumed to be 1.

- Calculate the limiting values:

$$m_{lim} = \beta\left(\frac{x}{d}\right)_{lim}\left[1 - \frac{\beta}{2}\left(\frac{x}{d}\right)_{lim}\right],$$

$$\omega_{lim} = \beta\left(\frac{x}{d}\right)_{lim},$$

$$a_{max} = \omega_{lim} d,$$

- Calculate ω, a, and $\frac{x}{d}$ as follows:

  $$\omega = 1 - \sqrt{1 - 2m}\ , \text{ and}$$

  $$a = \omega d \le a_{max}.$$

  $$\frac{x}{d} = \frac{\omega}{\beta}.$$

- If $a \le h_f$, the neutral axis lies within the flange. Calculate the area of tension reinforcement, A_s, as follows:

  If $m \le m_{lim}$,

  $$\omega = 1 - \sqrt{1 - 2m}\ , \text{ and}$$

  $$A_s = \omega \left[\frac{\alpha f_{cd} b_w d}{f_{cd}} \right].$$

  If $m > m_{lim}$,

  $$\omega' = \frac{m - m_{lim}}{1 - d'/d},$$

  $$\omega_{lim} = \beta \left(\frac{x}{d} \right)_{lim},$$

  $$\omega = \omega_{lim} + \omega',$$

  $$A_s = \omega, \left[\frac{\alpha f_{cd} b_w d}{f_{yd}} \right] \text{ and}$$

  $$A_s' = \omega' \left[\frac{\alpha f_{cd} b\ d}{f_s'} \right], \text{ where}$$

$$f_s' = E_s \varepsilon_c \left[1 - \frac{d'}{x_{lim}} \right] \leq f_{yd}. \qquad \text{(EC2 4.2.2.3.2)}$$

- If $a > h_f$, the neutral axis lies below the flange. Calculate the steel area required for equilibrating the flange compression, A_{s2}.

  $$A_{s2} = \frac{\left(b_f - b_w\right) h_f \alpha f_{cd}}{f_{yd}},$$

  and the corresponding resistive moment is given by

  $$M_2 = A_{s2} f_{yd} \left(d - \frac{h_f}{2} \right).$$

  Calculate the steel area required for the rectangular section of width b_w to resist moment, $M_1 = M - M_2$, as follows:

  $$m_1 = \frac{M_1}{b_w d^2 \alpha f_{cd}}, \text{ and}$$

  If $m_1 \leq m_{lim}$,

  $$\omega_1 = 1 - \sqrt{1 - 2m_1} \text{ , and}$$

  $$A_{s1} = \omega_1 \left[\frac{\alpha f_{cd} b_w d}{f_{yd}} \right].$$

  If $m_1 > m_{lim}$,

  $$\omega' = \frac{m_1 - m_{\lim}}{1 - d'/d},$$

  $$\omega_{lim} = \beta \left(\frac{x}{d} \right)_{\lim},$$

  $$\omega_1 = \omega_{lim} + \omega',$$

$$A_s' = \omega' \left[\frac{\alpha f_{cd} b d}{f_s'} \right], \text{ and}$$

$$A_{s1} = \omega_1 \left[\frac{\alpha f_{cd} b_w d}{f_{yd}} \right],$$

where, f_s' is given by

$$f_s' = E_s \varepsilon_c \left[1 - \frac{d'}{x_{lim}} \right] \le f_{yd}. \qquad \text{(EC2 4.2.2.3.2)}$$

Calculate total steel area required for the tension side.

$$A_s = A_{s1} + A_{s2}.$$

7.4.1.2.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel required for a beam section is given by the following equation (EC2 5.4.2.1.1):

$$A_s \ge \begin{cases} \dfrac{0.6}{f_{yk}} b d & \text{Rectangular beam} \\ \dfrac{0.6}{f_{yk}} b_w d & \text{T - beam.} \end{cases} \qquad \text{(EC2 5.4.2.1.1)}$$

In no case in the preceding equation, should the factor $0.6/f_{vk}$ be taken as less than 0.0015,

$$\frac{0.6}{f_{yk}} \ge 0.0015. \qquad \text{(EC2 5.4.2.1.1)}$$

The applicable minimum flexural tension reinforcement for control of cracking (EC2 4.4.2) should be investigated independently by the user.

An upper limit on the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (EC 5.4.2.1.1).

7.4.1.3 Design Beam Shear Reinforcement

The shear reinforcement is designed for each loading combination at the two stations at the ends of the beam elements. The assumptions in designing the shear reinforcement are as follows:

- The beam sections are assumed to be prismatic. The effect of any variation of width in the beam section on the concrete shear capacity is neglected.
- The effect on the concrete shear capacity of any concentrated or distributed load in the span of the beam between two columns is ignored. Also, the effect of the direct support on the beams provided by the columns is ignored.
- All shear reinforcement is assumed to be perpendicular to the longitudinal reinforcement.
- The effect of any torsion is neglected for the design of shear reinforcement.

In designing the shear reinforcement for a particular beam for a particular loading combination, the following steps of the standard method are involved.

- Obtain the design value of the applied shear force, V_{Sd}, from the SAFE analysis results.
- Calculate the design shear resistance of the member without shear reinforcement.

$$V_{Rd1} = \beta[\tau_{Rd} k(1.2 + 40\ \rho_l)]\ (b_w d), \text{ where} \qquad \text{(EC2 4.3.2.3)}$$

β = enhancement factor for shear resistance for members with concentrated loads located near the face of the support. β is taken as 1.

τ_{Rd} = basic design shear strength of concrete $= 0.25 f_{ctk\ 0.05} / \gamma_c$,

$f_{ctk\ 0.05} = 0.7\, f_{ctm}$, (EC2 3.1.2.3)

$f_{ctm} = 0.3\, f_{ck}^{2/3}$, (EC2 3.1.2.3)

k = strength magnification factor for curtailment of longitudinal reinforcement and is considered to be 1,

ρ_1 = tension reinforcement ratio = $\frac{A_{s1}}{b_w d} \le 0.02$, and

A_{s1} = area of tension reinforcement.

- Calculate the maximum design shear force that can be carried without crushing of the notional concrete compressive struts, V_{Rd2}.

$$V_{Rd2} = \frac{1}{2} \nu \left[\frac{f_{ck}}{\gamma_c} \right] (0.9 b_w d), \text{ where} \qquad \text{(EC2 4.3.2.3)}$$

ν is the effectiveness factor = $0.7 - \frac{f_{cd}}{200} \ge 0.5$. (EC2 4.3.2.3)

- Given V_{Sd}, V_{Rd1}, and $V_{Rd2,red}$, the required shear reinforcement in area/unit length is calculated as follows:

If $V_{Sd} \le V_{Rd1}$,

$$\frac{A_{sw}}{s_v} = 0, \qquad \text{(EC2 4.3.2.3)}$$

else if $V_{Rd1} < V_{Sd} \le V_{Rd2}$,

$$\frac{A_{sw}}{s_v} = \frac{(V_{Sd} - V_{Rd1})\gamma_s}{0.9\, d\, f_{ywk}} \qquad \text{(EC2 4.3.2.4.3)}$$

else if $V_{Sd} > V_{Rd2}$,

a failure condition is declared. (EC2 4.3.2.2)

An upper limit is imposed on the steel tensile strength:

$f_{ywk} / \gamma_s \le$ MPa (EC2 4.3.2.2)

The maximum of all the calculated A_{sw}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

A lower limit is imposed on A_{sw}/s:

$$\frac{A_{sw}}{s_v} \geq \rho_{w,min} b_w \qquad \text{(EC2 5.4.2.2)}$$

where $\rho_{w,min}$ is obtained from the following table (EC2 Table 7.5):

Minimum Values of Shear Stress Ratio, $\rho_{w,min}$ (EC2 5.4.2.2, EC2 Table 7.5)			
Concrete Strength	$f_y \leq 220$	$220 < f_y \leq 400$	$f_y > 400$
$f_c' \leq 20$	0.0016	0.0009	0.0007
$20 < f_c' \leq 35$	0.0024	0.0013	0.0011
$f_c' > 35$	0.0030	0.0016	0.0013

The beam shear reinforcement requirements displayed by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

7.5 Slab Design

As is done in conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method (Eurocode 2) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

7.5.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Design flexural reinforcement for the strip

These two steps, described in the text that follows, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

7.5.1.1 Determine Factored Moments for Strip

For each element within the design strip, for each load combination the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

7.5.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip given the bending moment is identical to the design of rectangular beam sections described earlier. When the slab properties (depth, etc.) vary over the width of the strip, the program automatically designs slab widths of each property separately for the bending moment to which they are subjected and then sums the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

7.5.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement required in each direction of a slab is given by the following limits:

$$A_s \geq \begin{cases} \dfrac{0.6}{f_y} bd \\ 0.0015bd \end{cases} \qquad \text{(EC 2.5.4.2.1.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (EC2 5.4.2.1.1).

7.5.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the following sections.

7.5.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of 1.5*d* from the face of the support (EC2 4.3.4.2.2).

7.5.2.2 Determination of Concrete Capacity

The factored concrete punching shear strength is taken as the design shear resistance per unit length without shear reinforcement.

$$\nu_{Rd1} = [\tau_{Rd} k(1.2 + 40\ \rho_1)]\ d, \text{ where} \qquad \text{(EC2 4.3.4.5.1)}$$

$$\tau_{Rd} = \text{basic design shear strength} = \frac{0.25 f_{ctk0.05}}{\gamma_c}, \qquad \text{(EC2 4.3.2.3)}$$

$$f_{ctk0.05} = 0.7 f_{ctm}, \qquad \text{(EC2 3.1.2.3)}$$

$$f_{ctm} = 0.3\ f_{ck}^{2/3}, \qquad \text{(EC2 3.1.2.3)}$$

$$k = 1.6 - \frac{d}{1000} \geq 1.0 \text{ , } d \text{ in mm} \qquad \text{(EC2 4.3.4.5.1)}$$

$$\rho_1 = \sqrt{\rho_{1x}\rho_{1y}} \leq 0.015$$

$$d = \frac{d_x + d_y}{2},$$

ρ_{1x} and ρ_{1y} are the reinforcement ratios in the X and Y directions, respectively, conservatively taken as zeros, and

d_x and d_y are the effective depths of the slab at the points of intersection between the design failure surface and the longitudinal reinforcement, in the X and Y directions, respectively.

7.5.2.3 Determination of Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the factored punching shear force per unit length is taken as follows:

$$\nu_{Sd} = \frac{V_{Sd}\beta}{u}, \text{ where} \qquad \text{(EC2 4.3.4.3)}$$

ν_{Sd} is the total design shear force developed,

u is the perimeter of the critical section, and

β is the coefficient that accounts for the effects of eccentricity of loading

$$\beta = \begin{cases} 1.15 & \text{for interior columns} \\ 1.40 & \text{for edge columns, and} \\ 1.50 & \text{for corner columns.} \end{cases} \qquad \text{(EC2.4.3.4.3)}$$

The ratio of the maximum factored shear force and the concrete punching shear resistance is reported by SAFE.

Chapter 8
Design for NZS 3101-95

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the New Zealand code, NZS 3101-95 (NZS 1995). Various notations used in this chapter are listed in Table 8-1. For referencing to the pertinent sections of the New Zealand code in this chapter, a prefix "NZS" followed by the section number is used here.

The design is based on user-specified loading combinations. However, the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-Millimeter-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-Millimeter-Second units unless otherwise noted.

8.1 Notations

Table 8-1 List of Symbols Used in the New Zealand Code

A_{cv}	Area of concrete used to determine shear stress, sq-mm
A_g	Gross area of concrete, sq-mm

Table 8-1 List of Symbols Used in the New Zealand Code

A_s	Area of tension reinforcement, sq-mm
A_s'	Area of compression reinforcement, sq-mm
$A_{s(required)}$	Area of steel required for tension reinforcement, sq-mm
A_v	Area of shear reinforcement, sq-mm
A_v/s	Area of shear reinforcement per unit length of the member, sq-mm/mm
s	Spacing of shear reinforcement along the length of the beam, mm
a	Depth of compression block, mm
a_b	Depth of compression block at balanced condition, mm
a_{max}	Maximum allowed depth of compression block, mm
b	Width of member, mm
b_f	Effective width of flange (T-Beam section), mm
b_w	Width of web (T-Beam section), mm
b_0	Perimeter of the punching critical section, mm
b_1	Width of the punching critical section in the direction of bending, mm
b_2	Width of the punching critical section perpendicular to the direction of bending, mm
c	Depth to neutral axis, mm
c_b	Depth to neutral axis at balanced conditions, mm
d	Distance from compression face to tension reinforcement, mm
d'	Concrete cover to center of reinforcing, mm
d_s	Thickness of slab (T-Beam section), mm
E_c	Modulus of elasticity of concrete, MPa

Table 8-1 List of Symbols Used in the New Zealand Code

E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_c'	Specified compressive strength of concrete, PMa ($17.5 \le f_c' \le 100$)
f_y	Specified yield strength of flexural reinforcement, MPa ($f_y \le 500$)
f_{yt}	Specified yield strength of shear reinforcement, MPa ($f_t \le 800$)
h	Overall thickness of slab or overall depth of a beam, mm
M^*	Factored moment of section, N-mm
V_c	Shear resisted by concrete, N
V_{max}	Maximum permitted total factored shear force at a section, lb
V^*	Factored shear force at a section, N
V_s	Shear force at a section resisted by steel, N
v	Average design shear stress at a section, MPa
v_b	Basic design shear stress resisted by concrete, MPa
v_c	Design shear stress resisted by concrete, MPa
v_{max}	Maximum design shear stress permitted at a section, MPa
α_1	Concrete strength factor to account for sustained loading and equivalent stress block
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
ε_s	Strain in reinforcing steel
φ_b	Strength reduction factor for bending
φ_s	Strength reduction factor for shear

Table 8-1 List of Symbols Used in the New Zealand Code

γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear

8.2 Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (DL), live load (LL), pattern live load (PLL), wind (WL), and earthquake (EL) loads, and considering that wind and earthquake forces are reversible, then the following load combinations must be considered (NZS 4203-92 2.4.3):

1.4 DL

1.2 DL + 1.6 LL (NZS 4203-92 2.4.3.3)

1.2 DL + 1.6*0.75 PLL (NZS 3101-95 14.9.6.3)

1.2 DL ± 1.0 WL
0.9 DL ±1.0 WL
1.2 DL + 0.4 LL ± 1.0 WL (NZS 4203-92 2.4.3.3)

1.0 DL ± 1.0 EL
1.0 DL + 0.4 LL ± 1.0 EL (NZS 4203-92 2.4.3.3)

These are also the default design load combinations in SAFE whenever the NZS 3101-95 code is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

8.3 Strength Reduction Factors

The default strength reduction factor, φ, is taken as

φ_b = 0.85 for bending, (NZS 3.4.2.2)

$\varphi_s = 0.75$ for shear. (NZS 3.4.2.2)

The user is allowed to overwrite these values. However, caution is advised.

8.4 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on the beam moments, shear forces, load combination factors and other criteria described in the sections that follow. The reinforcement requirements are calculated at two check stations at the ends of the beam elements. All the beams are designed for major direction flexure and shear only. Effects due to any axial forces, minor direction bending, and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design beam flexural reinforcement
- Design beam shear reinforcement

8.4.1 Design Beam Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the ends of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine the maximum factored moments
- Determine the reinforcing steel

8.4.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all the of the load combinations. Positive beam moments produce bottom steel. In such cases, the beam may be designed as a Rectangular or a T-beam. Negative beam moments produce top steel. In such cases, the beam is always designed as a rectangular section.

8.4.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block as shown in Figure 8-1 (NZS 8.3.1.6). Furthermore, it is assumed that the compression carried by concrete is 0.75 times that which can be carried at the balanced condition (NZS 8.4.2). When the applied moment exceeds the moment capacity at the balanced condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

In designing the beam flexural reinforcement, the following limits are imposed on the steel tensile strength and the concrete compressive strength:

$$f_y \leq 500 \text{ MPa} \qquad \text{(NZS 3.8.2.1)}$$

$$f_c' \leq 100 \text{ MPa} \qquad \text{(NZS 3.8.1.1)}$$

The design procedure used by SAFE for both rectangular and flanged sections (L- and T-beams) is summarized in the sections that follow. All the beams are designed for major direction flexure and shear only.

8.4.1.2.1 Design for Flexure of a Rectangular Beam

In designing for a factored negative or positive moment, M^* (i.e., designing top or bottom steel), the depth of the compression block, a (see Figure 8-1), is computed as,

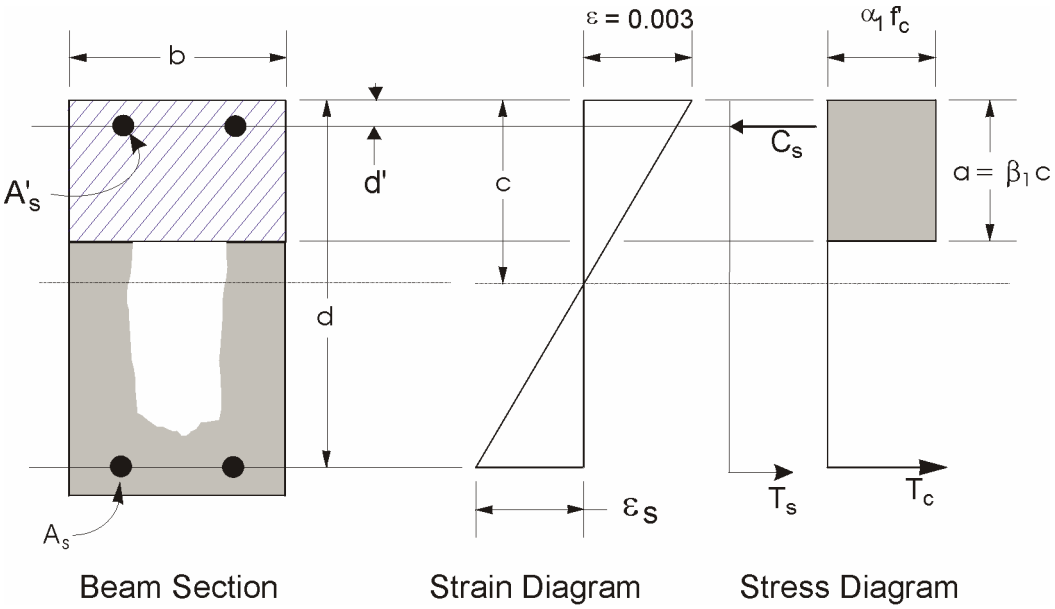


Figure 8-1 Design of a Rectangular Beam Section

$$a = d - \sqrt{d^2 - \frac{2\left|M^*\right|}{\alpha_1 f'_c \varphi_b b}}, \qquad \text{(NZS 8.3.1)}$$

where the default value of φ_b is 0.85 (NZS 3.4.2.2) in the preceding and following equations. Also, α_1 is calculated as follows:

$$\alpha_1 = 0.85 - 0.004\,(f'_c - 55), \quad 0.75 \le \alpha_1 \le 0.85. \qquad \text{(NZS 8.3.1.7)}$$

Also β_1 and c_b are calculated as follows:

$$\beta_1 = 0.85 - 0.008\,(f'_c - 30), \quad 0.65 \le \beta_1 \le 0.85, \text{ and} \qquad \text{(NZS 8.3.1.7)}$$

$$c_b = \frac{600}{600 + f_y} d. \qquad \text{(NZS 8.4.1.2)}$$

The maximum allowed depth of the compression block is given by

$$a_{max} = 0.75\beta_1 c_b. \qquad \text{(NZS 8.4.2 and NZS 8.3.1.7)}$$

- If $a \le a_{max}$ (NZS 8.4.2), the area of tensile steel reinforcement is then given by

$$A_s = \frac{M^*}{\varphi_b f_y \left(d - \frac{a}{2} \right)}.$$

This steel is to be placed at the bottom if M^* is positive, or at the top if M^* is negative.

- If $a > a_{max}$ (NZS 8.4.2), compression reinforcement is required (NZS 8.4.1.3) and is calculated as follows:

  − The compressive force developed in concrete alone is given by

$$C = \alpha_1 f_c' b a_{\max}, \text{ and} \qquad \text{(NZS 8.3.1.7)}$$

the moment resisted by concrete and bottom steel is

$$M^*_c = C\left(d - \frac{a_{max}}{2} \right) \varphi_b.$$

The moment resisted by compression steel and tensile steel is

$$M^*_s = M^* - M^*_c.$$

So the required compression steel is given by

$$A_s' = \frac{M^*_s}{\left(f_s^{'} - \alpha_1 f_c^{'}\right)\left(d - d'\right)\varphi_b}, \text{ where}$$

$$f_s' = 0.003 E_s \left[\frac{c - d'}{c} \right] \le f_y. \qquad \text{(NZS 8.3.1.2 and NZS 8.3.1.3)}$$

The required tensile steel for the balancing the compression in concrete is

$$A_{s1} = \frac{M_c^*}{f_y\left(d - \frac{a_{max}}{2}\right)\varphi_b}, \text{ and}$$

the tensile steel for balancing the compression in steel is

$$A_{s2} = \frac{M_s^*}{f_y(d - d')\varphi_b}.$$

Therefore, the total tensile reinforcement, $A_s = A_{s1} + A_{s2}$, and total compression reinforcement is A_s'. A_s is to be placed at the bottom and A_s' is to be placed at the top if M^* is positive, and vice versa.

8.4.1.2.2 Design for Flexure of a T-Beam

(i) Flanged Beam Under Negative Moment

In designing for a factored negative moment, M^* (i.e., designing top steel), the calculation of the steel area is exactly the same as above, i.e., no T-Beam data is to be used.

(ii) Flanged Beam Under Positive Moment

If $M^* > 0$, the depth of the compression block is given by (see Figure 8-2),

$$a = d - \sqrt{d^2 - \frac{2|M^*|}{\alpha_1 f_c' \varphi_b b_f}}. \qquad \text{(NZS 8.3.1)}$$

The maximum allowed depth of the compression block is given by

$$a_{max} = 0.75\ \beta_1 c_b. \qquad \text{(NZS 8.4.2 and NZS 8.3.1.7)}$$

- If $a \le d_s$ (NZS 8.4.2), the subsequent calculations for A_s are exactly the same as previously done for the rectangular section design. However, in this case, the width of the beam is taken as b_f. Whether compression reinforcement is required depends on whether $a > a_{max}$.
- If $a > d_s$ (NZS 8.4.2), calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is

for balancing the compressive force from the web, C_w. As shown in Figure 8-2,

$$C_f = \alpha 1\, f_c'\ (b_f - b_w)\ \min(d_s, a_{max}). \qquad \text{(NZS 8.3.1.7)}$$

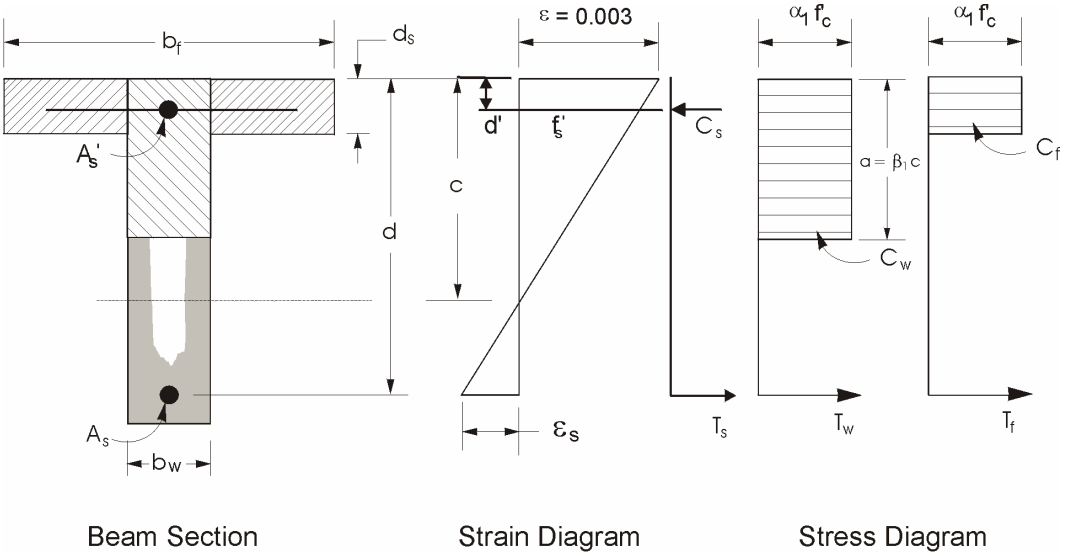


Figure 8-2 Design of a T-Beam Section

Therefore, $A_{s1} = \dfrac{C_f}{f_y}$ and the portion of M^* that is resisted by the flange is given by

$$M_f^* = C_f\left(d - \frac{\min(d_s, a_{max})}{2}\right)\varphi_b\,.$$

Therefore, the balance of the moment, M^* to be carried by the web is given by

$$M_w^* = M^* - M_f^*.$$

The web is a rectangular section of dimensions b_w and d, for which the depth of the compression block is recalculated as

$$a_1 = d - \sqrt{d^2 - \frac{2M_w^*}{\alpha_1 f_c' \varphi_b b_w}}. \qquad \text{(NZS 8.3.1)}$$

If $a_1 \le a_{max}$ (NZS 8.4.2), the area of tensile steel reinforcement is then given by

$$A_{s2} = \frac{M_w^*}{\varphi_b f_y \left(d - \frac{a_1}{2} \right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2}.$$

This steel is to be placed at the bottom of the T-beam.

- If $a_1 > a_{max}$ (NZS 8.4.2), compression reinforcement is required and is calculated as follows:

  The compressive force in the concrete web alone is given by

  $$C_w = \alpha_1 f_c' b_w a_{max}, \text{ and} \qquad \text{(NZS 8.3.1.7)}$$

  the moment resisted by the concrete web and tensile steel is

  $$M_c^* = C_w \left(d - \frac{a_{max}}{2} \right) \varphi_b.$$

  The moment resisted by compression steel and tensile steel is

  $$M_s^* = M_w^* - M_c^*.$$

  Therefore, the compression steel is computed as

  $$A_s' = \frac{M_s^*}{\left(f_s' - \alpha_1 f_c' \right) \left(d - d' \right) \varphi_b}, \text{ where}$$

$$f_s' = 0.003E_s\left[\frac{c-d'}{c}\right] \le f_y. \qquad \text{(NZS 8.3.1.2 and NZS 8.3.1.3)}$$

The tensile steel for balancing compression in web concrete is

$$A_{s2} = \frac{M_c^*}{f_y\left(d - \frac{a_{max}}{2}\right)\varphi_b}, \text{ and}$$

the tensile steel for balancing compression in steel is

$$A_{s3} = \frac{M_s^*}{f_y\left(d-d'\right)\varphi_b}.$$

Total tensile reinforcement, $A_s = A_{s1} + A_{s2} + A_{s3}$, and total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top.

8.4.1.2.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel required for a beam section is given by the minimum of the two limits:

$$A_s \ge \frac{\sqrt{f_c'}}{4f_y}\, b_w d, \quad \text{or} \qquad \text{(NZS 8.4.3.1)}$$

$$A_s \ge \frac{4}{3}A_{s(required)}. \qquad \text{(NZS 8.4.3.3)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{T - beam} \end{cases}$$

$$A_s' \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{T - beam} \end{cases}$$

8.4.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at two stations at the ends of each beam element. In designing the shear reinforcement of a particular beam for a particular loading combination at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V^*.
- Determine the shear force, V_c, that can be resisted by the concrete.
- Determine the reinforcement steel required to carry the balance.

In designing the beam shear reinforcement, the following limits are imposed on the steel tensile strength and the concrete compressive strength:

$$f_{yt} \leq 500 \text{ MPa} \qquad \text{(NZS 3.8.2.1 and NZS 9.3.6.1)}$$

$$f_c' \leq 100 \text{ MPa} \qquad \text{(NZS 3.8.1.1)}$$

The following three sections describe in detail the algorithms associated with the preceding steps.

8.4.2.1 Determine Shear Force and Moment

In the design of the beam shear reinforcement of a concrete frame, the shear forces and moments for a particular load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the corresponding load combination factors.

8.4.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as follows:

The basic shear strength for rectangular section is computed as,

$$\nu_b = \left[0.07 + 10 \frac{A_s}{b_w d} \right] \sqrt{f_c'} \text{ , where} \qquad \text{(NZS 9.3.2.1)}$$

$$\sqrt{f_c'} \le \sqrt{70}\ , \text{ and} \qquad \text{(NZS 9.3.2.1)}$$

$$0.08\sqrt{f_c'} \le \nu_b \le 0.2\sqrt{f_c'}. \qquad \text{(NZS 9.3.2.1)}$$

The allowable shear capacity is given by,

$$\nu_c = \nu_b. \qquad \text{(NZS 9.3.2.1)}$$

8.4.2.3 Determine Required Shear Reinforcement

The average shear stress is computed for a rectangular section as,

$$\nu^* = \frac{V^*}{b_w d}. \qquad \text{(NZS 9.3.1.1)}$$

The average shear stress is limited to a maximum limit of,

$$v_{max} = \min\left\{1.1\sqrt{f_c'}, 0.2 f_c', 9\text{MPa}\right\}. \qquad \text{(NZS 9.3.1.8)}$$

The shear reinforcement is computed as follows:

- If $\nu^* \le \varphi_s \left(v_c/2\right)$,

$$\frac{A_v}{s} = 0, \qquad \text{(NZS 9.3.4.1)}$$

else if $\varphi_s \left(v_c/2\right) < \nu^* \le \varphi_s (\nu_c + 0.35)$,

$$\frac{A_v}{s} = \frac{0.35b}{f_{yt}}, \qquad \text{(NZS 9.3.4.3)}$$

else if $\varphi_s (\nu_c + 0.35) < \nu^* \le \varphi_s \nu_{max}$, (NZS 9.3.6.3)

$$\frac{A_v}{s} = \frac{\left(v^* - \varphi_s v_c\right) b_w}{\varphi_s f_{yt}}$$

else if $\nu^* > \nu_{max}$,

a failure condition is declared. (NZS 9.3.1.8)

The maximum of all the calculated A_s/s values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

The beam shear reinforcement requirements displayed by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

8.5 Slab Design

As is done in conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method for reinforced concrete (NZS 3101-95) as described in the following sections. To learn more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

8.5.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip

- Design flexural reinforcement for the strip

These two steps, described in the sections that follow, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

8.5.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

8.5.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip given the bending moment is identical to the design of rectangular beam sections described earlier. Where the slab properties (depth, etc.) vary over the width of the strip, the program automatically designs slab widths of each property separately for the bending moment they are subjected to before summing up the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

8.5.1.3 Minimum Slab Reinforcement

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limit (NZS 8.4.3.4):

$$A_s \geq \begin{cases} \dfrac{0.7}{f_y} bh & \text{if } f_y < 500 MPa \\ 0.0014 bh & \text{if } f_s \geq 500 MPa. \end{cases} \qquad \text{(NZS 7.3.30.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

8.5.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the sections that follow.

8.5.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of *d*/2 from the face of the support (NZS 9.3.15.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (NZS 9.3.15.1).

8.5.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M^*$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M^*$, where

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \text{ and} \qquad \text{(NZS 14.3.5)}$$

$$\gamma_v = 1 - \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \qquad \text{(NZS 9.3.16.2)}$$

where b_1 is the width of the critical section measured in the direction of the span, and b_2 is the width of the critical section measured in the direction perpendicular to the span.

8.5.2.3 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$v_c = min\begin{cases} \varphi_s\left(1+\beta_c\right)0.17\sqrt{f_c'} \\ \varphi_s\left(1+\dfrac{a_s d}{2b_o}\right)0.17\sqrt{f_c'} \\ \varphi_s 0.33\sqrt{f_c'} \end{cases} \qquad \text{(NZS 9.3.15.3)}$$

where, β_c is the ratio of the minimum to the maximum dimensions of the critical section, b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 40 & \text{for interior columns,} \\ 30 & \text{for edge columns, and} \\ 20 & \text{for corner columns.} \end{cases} \qquad \text{(NZS 9.3.15.2)}$$

A limit on $\sqrt{f_c'}$ is imposed as follows:

$$\sqrt{f_c'} \le \sqrt{70}. \qquad \text{(NZS 9.3.2.1)}$$

8.5.2.4 Determination of Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by SAFE.

Chapter 9
Design for IS 456-78 (R1996)

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the Indian Code IS 345-78 Revision 1996 (IS 1996). Various notations used in this chapter are listed in Table 9-1. For referencing to the pertinent sections of the Indian code in this chapter, a prefix "IS" followed by the section number is used here.

The design is based on user-specified loading combinations. However, the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-Millimeter-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-Millimeter-Second units unless otherwise noted.

9.1 Notations

Table 9-1 List of Symbols Used in the Indian Code

A_c	Area of concrete, mm^2

Table 9-1 List of Symbols Used in the Indian Code

A_{cv}	Area of section for shear resistance, mm^2
A_g	Gross cross-sectional area of a frame member, mm^2
A_s	Area of tension reinforcement, mm^2
A_s'	Area of compression reinforcement, mm^2
A_{sv}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sv}/s_v	Area of shear reinforcement per unit length of the member, mm^2/mm
a_1	Width of the punching critical section in the direction of bending, mm
a_2	Width of the punching critical section perpendicular to the direction of bending, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d	Effective depth of tension reinforcement, mm
d'	Effective depth of compression reinforcement, mm
$d_{compression}$	Depth of center of compression block from most compressed face, mm
D	Overall depth of a beam or slab, mm
D_f	Flange thickness in a T-beam, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_{cd}	Design concrete strength = f_{ck}/γ_c, MPa
f_{ck}	Characteristic compressive strength of concrete, MPa

Table 9-1 List of Symbols Used in the Indian Code

$f_s^{'}$	Compressive stress in beam compression steel, MPa
f_{yd}	Design yield strength of reinforcing steel = f_y / γ_s, MPa
f_y	Characteristic strength of reinforcement, MPa
f_{ys}	Characteristic strength of shear reinforcement, MPa
h	Overall thickness of slab, mm
k	Enhancement factor of shear strength for depth of the beam
M_{single}	Design moment resistance of a section as a singly reinforced section, N-mm
M_u	Ultimate factored design moment at a section, N-mm
m	Normalized design moment, $M / db^2 \alpha f_{ck}$
s_v	Spacing of the shear reinforcement along the length of the beam, mm
V_u	Shear force of ultimate design load, N
v_c	Allowable shear stress in punching shear mode, N
x_u	Depth of neutral axis, mm
$x_{u,max}$	Maximum permitted depth of neutral axis, mm
Z	Lever arm, mm
α	Concrete strength reduction factor for sustained loading; also fraction of moment to be transferred by flexure in a slab-column joint
β	Factor for the depth of compressive force resultant of the concrete stress block
β_c	Ratio of the maximum to minimum dimensions of the punching critical section
γ_c	Partial safety factor for concrete strength

Table 9-1 List of Symbols Used in the Indian Code

γ_f	Partial safety factor for load, and fraction of unbalanced moment transferred by flexure
γ_m	Partial safety factor for material strength
γ_s	Partial safety factor for steel strength
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear
δ	Enhancement factor of shear strength for compression
$\varepsilon_{c,max}$	Maximum concrete strain in the beam and slab (= 0.0035)
ε_s	Strain in tension steel
ε_s'	Strain in compression steel
ρ	Tension reinforcement ratio, A_s/bd
τ_v	Average design shear stress resisted by concrete, MPa
τ_c	Basic design shear stress resisted by concrete, MPa
$\tau_{c,max}$	Maximum possible design shear stress permitted at a section, MPa
τ_{cd}	Design shear stress resisted by concrete, MPa

9.2 Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (DL), live load (LL), pattern live load (PLL), wind (WL), and earthquake (EL) loads, and considering that wind and earthquake forces are reversible, then the following load combinations must be considered (IS 35.4):

1.5 DL
1.5 DL + 1.5 LL (IS 35.4.1)

1.5 DL + 1.5*0.75 PLL (IS 30.5.2.3)
1.5 DL ± 1.5 WL
0.9 DL ± 1.5 WL
1.2 DL + 1.2 LL ± 1.2 WL (IS 35.4.1)

1.5 DL ±1.5 EL
0.9 DL ± 1.5 EL
1.2 DL + 1.2 LL ±1.2 EL (IS 35.4.1)

These are also the default design load combinations in SAFE whenever the Indian Code is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

9.3 Design Strength

The design strength for concrete and steel are obtained by dividing the characteristic strength of the material by a partial factor of safety, γ_m. The values of γ_m used in the program are listed as follows:

- Partial safety factor for steel, $\gamma_s = 1.15$, and (IS 35.4.2.1)
- Partial safety factor for concrete, $\gamma_c = 1.5$. (IS 35.4.2.1)

These factors are already incorporated in the design equations and tables in the code. SAFE does not allow them to be overwritten.

9.4 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on the beam moments, shears, load combination factors, and other criteria described in the sections that follow. The reinforcement requirements are calculated at two check stations at the ends of the beam elements.

All the beams are designed for major direction flexure and shear only. Effects due to any axial forces, minor direction bending, and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design beam flexural reinforcement
- Design beam shear reinforcement

9.4.1 Design Beam Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the ends of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine the maximum factored moments
- Determine the reinforcing steel

9.4.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all the of the load combinations. Positive beam moments produce bottom steel. In such cases the beam may be designed as a Rectangular or a T-beam. Negative beam moments produce top steel. In such cases the beam is always designed as a rectangular section.

9.4.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified parabolic stress block as shown in Figure 9-1 (IS 37.1). The area of the stress block, *C*, and the depth of the center of the compressive force from the most compressed fiber, *d*, are taken as

$$C = \alpha f_{ck} x_u \text{ and} \qquad \text{(IS 37.1)}$$

$$d_{compression} = \beta x_u, \qquad \text{(IS 37.1)}$$

where x_u is the depth of the compression block, and α and β are taken respectively as

$$\alpha = 0.36, \text{ and} \qquad \text{(IS 37.1)}$$

$$\beta = 0.42. \qquad \text{(IS 37.1)}$$

α is the reduction factor to account for sustained compression and the partial safety factor for concrete. α is generally assumed to be 0.36 for the assumed parabolic stress block (IS 37.1). The β factor considers the depth of the neutral axis.

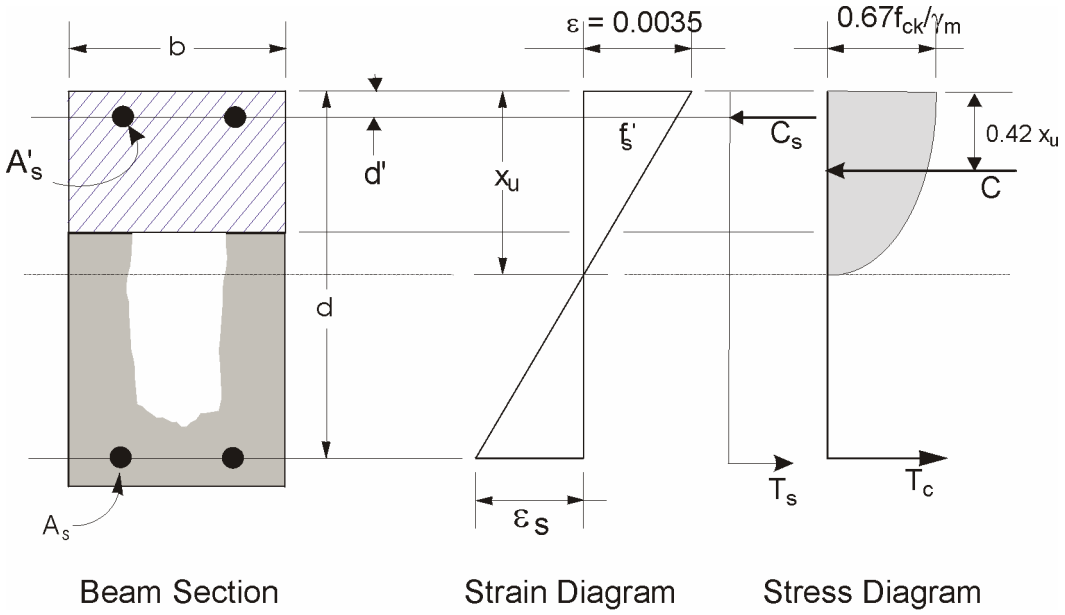


Figure 9-1 Design of a Rectangular Beam Section

Furthermore, it is assumed that moment redistribution in the member does not exceed the code specified limiting value. The code also places a limitation on the neutral axis depth as shown below, to safeguard against non-ductile failures (IS 37.1).

f_y	$x_{u,max}/d$
250	0.53
415	0.48
500	0.46

SAFE uses interpolation between the three discrete points given in the code.

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \\ 0.53 - 0.05\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \le 415 \\ 0.48 - 0.02\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \le 500 \\ 0.46 & \text{if} \quad f_y \ge 500 \end{cases} \qquad \text{(IS 37.1)}$$

When the applied moment exceeds the capacity of the beam as a singly reinforced beam, the area of compression reinforcement is calculated on the assumption that the neutral axis depth remains at the maximum permitted value. The maximum fiber compression is taken as

$$\varepsilon_{c,max} = 0.0035, \qquad \text{(IS 37.1)}$$

and the modulus of elasticity of steel is taken to be

$$E_s = 200{,}000 \text{ MPa} . \qquad \text{(IS 37.1)}$$

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams) is summarized in the following sections. It is assumed that the design ultimate axial force can be neglected; hence all the beams are designed for major direction flexure and shear only.

9.4.1.2.1 Design as a Rectangular Beam

For rectangular beams, the limiting depth of neutral axis, $x_{u,max}$, and the moment capacity as a singly reinforced beam, M_{single}, are obtained first for the section. The reinforcing steel area is determined based on whether M_u is greater than, less than, or equal to M_{single}. See Figure 9-1.

- Calculate the limiting depth of the neutral axis.

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \\ 0.53 - 0.05\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \le 415 \\ 0.48 - 0.02\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \le 500 \\ 0.46 & \text{if} \quad f_y \ge 500 \end{cases} \qquad \text{(IS 37.1)}$$

- Calculate the limiting ultimate moment of resistance as a singly reinforced beam.

$$M_{single} = \alpha f_{ck} b d^2 \ \frac{x_{u,\max}}{d}\left[1 - \beta\frac{x_{u,\max}}{d}\right], \text{ where} \qquad \text{(IS E-1.1)}$$

$$\alpha = 0.36, \quad \text{and} \qquad \text{(IS E-1.1)}$$

$$\beta = 0.42. \qquad \text{(IS E-1.1)}$$

- Calculate the depth of neutral axis x_u as

$$\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4\beta m}}{2\beta},$$

where the normalized design moment, *m,* is given by

$$m = \frac{M_u}{b d^2 \alpha f_{ck}}.$$

- If $M_u \le M_{single}$ the area of tension reinforcement, A_s, is obtained from

$$A_s = \frac{M_u}{\left(f_y / \gamma_s\right) z}, \text{ where} \qquad \text{(IS E-1.1)}$$

$$z = d\left\{1 - \beta \frac{x_u}{d}\right\}. \qquad \text{(IS 37.1)}$$

This is the top steel if the section is under negative moment and the bottom steel if the section is under positive moment.

- If $M_u > M_{single}$, the area of compression reinforcement, A_s', is given by

$$A_s' = \frac{M_u - M_{single}}{f_s'\left(d - d'\right)}, \qquad \text{(IS E-1.2)}$$

where d' is the depth of the compression steel from the concrete compression face, and

$$f_s' = \varepsilon_{c,max} E_s \left[1 - \frac{d'}{x_{u,\max}}\right] \le \frac{f_y}{\gamma_s}. \qquad \text{(IS E-1.2)}$$

This is the bottom steel if the section is under negative moment. From equilibrium, the area of tension reinforcement is calculated as

$$A_s = \frac{M_{single}}{\left(f_y / \gamma_s\right) z} + \frac{M_u - M_{single}}{\left(f_y / \gamma_s\right)\left(d - d'\right)}, \text{ where} \qquad \text{(IS E-1.2)}$$

$$z = d\left\{1 - \beta \frac{x_{u,max}}{d}\right\}. \qquad \text{(IS 37.1)}$$

9.4.1.2.2 Design as a T-Beam

(i) Flanged Beam Under Negative Moment

The contribution of the flange to the strength of the beam is ignored if the flange is in the tension side. See Figure 9-2. The design procedure is therefore identical to the one used for rectangular beams. However, the width of the web, b_w, is taken as the width of the beam.

(ii) Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially the neutral axis is assumed to be located within the flange. Based on this assumption, the program calculates the depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width $b_{f.}$ If the stress block extends beyond the flange, additional calculation is required. See Figure 9-2.

- Assuming the neutral axis to lie in the flange, calculate the depth of neutral axis, x_u, as

$$\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4\beta m}}{2\beta},$$

where the normalized design moment, m, is given by

$$m = \frac{M_u}{b_f d^2 \alpha f_{ck}}.$$

- If $\left(\frac{x_u}{d}\right) \leq \left(\frac{D_f}{d}\right)$, the neutral axis lies within the flange. The subsequent calculations for A_s are exactly the same as previously defined for the rectangular section design (IS E-2.1). However, in this case the width of the compression flange, b_f, is taken as the width of the beam, b, for analysis. Whether compression reinforcement is required depends on whether $M_u > M_{single}$.

- If $\left(\frac{x_u}{d}\right) > \left(\frac{D_f}{d}\right)$, the neutral axis lies below the flange. Then calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 9-2.

  – Calculate the ultimate resistance moment of the flange as

$$M_f = 0.45 f_{ck} (b_f - b_w) y_f (d - 0.5 y_f), \qquad \text{(IS E-2.2)}$$

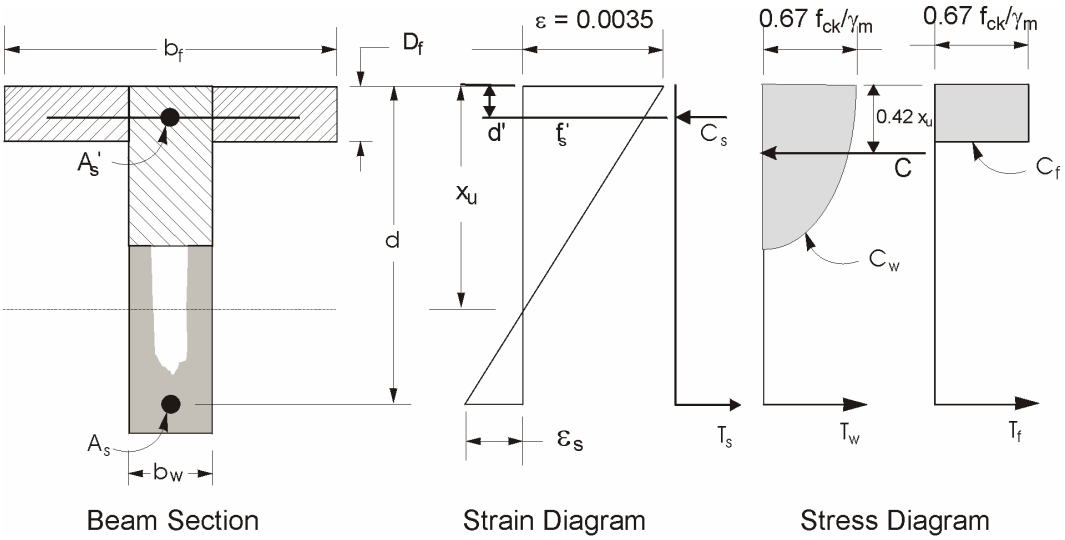


Figure 9-2 Design of a T-Beam Section

where y_f is taken as follows:

$$y_f = \begin{cases} D_f & \text{if } D_f \le 0.2d \\ 0.15x_u + 0.65D_f & \text{if } D_f > 0.2d \end{cases}$$

– Calculate the moment taken by the web as

$$M_w = M_u - M_f.$$

– Calculate the limiting ultimate moment of resistance of the web for tension only reinforcement.

$$M_{w,single} = \alpha f_{ck} b_w d^2 \frac{x_{u,\max}}{d}\left[1 - \beta \frac{x_{u,\max}}{d}\right] \text{ where} \qquad \text{(IS E-1.1)}$$

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \\ 0.53 - 0.05\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \le 415 \\ 0.48 - 0.02\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \le 500 \\ 0.46 & \text{if} \quad f_y \ge 500 \end{cases} \qquad \text{(IS 37.1)}$$

$$\alpha = 36\,, \quad \text{and} \qquad \text{(IS 37.1)}$$

$$\beta = 42. \qquad \text{(IS 37.1)}$$

– If $M_w \le M_{w,single}$, the beam is designed as a singly reinforced concrete beam. The area of steel is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.

$$A_s = \frac{M_f}{\left(f_y/\lambda_s\right)\left(d - 0.5\,y_f\right)} + \frac{M_w}{\left(f_y/\gamma_s\right)z}, \quad \text{where}$$

$$z = d\left\{1 - \beta\frac{x_u}{d}\right\},$$

$$\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4\beta m}}{2\beta}, \quad \text{and}$$

$$m = \frac{M_w}{b_w d^2 \alpha f_{ck}}.$$

– If $M_w > M_{w,single}$, the area of compression reinforcement, A'_s, is given by

$$A_s' = \frac{M_w - M_{w,single}}{f_s'\left(d - d'\right)},$$

where d' is the depth of the compression steel from the concrete compression face, and

$$f_s' = \varepsilon_{c,max} E_s . \left[1 - \frac{d'}{x_{u,max}}\right] \le \frac{f_y}{\gamma_s}. \qquad \text{(IS E-1.2)}$$

This is the bottom steel if the section is under negative moment. From equilibrium, the area of tension reinforcement is calculated as

$$A_s = \frac{M_f}{(f_y/\gamma_s)(d - 0.5y_f)} + \frac{M_{w,single}}{(f_y/\gamma_s)z} + \frac{M_w - M_{w,single}}{((f_y/\gamma_s)(d - d')},$$

where

$$z = d\left\{1 - \beta\frac{x_{u,max}}{d}\right\}.$$

9.4.1.2.3 Minimum Tensile Reinforcement

The minimum flexural tensile steel required for a beam section is given by the following equation (IS 25.5.1.1):

$$A_s \geq \begin{cases} \frac{0.85}{f_y}bd & \text{Rectangular beam} \\ \frac{0.85}{f_y}b_w d & \text{T - beam} \end{cases} \qquad \text{(IS 25.5.1.1)}$$

An upper limit on the tension reinforcement (IS 25.5.1.1) and compression reinforcement (IS 25.5.1.2) has been imposed to be 0.04 times the gross web area.

$$A_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{T - beam} \end{cases} \qquad \text{(IS 25.5.1.1)}$$

$$A_s' \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{T - beam} \end{cases} \qquad \text{(IS 25.5.1.2)}$$

9.4.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each loading combination at two stations at the ends of each beam element. The assumptions in designing the shear reinforcement are as follows:

- The beam sections are assumed to be prismatic. The effect of any variation of width in the beam section on the concrete shear capacity is neglected.
- The effect on the concrete shear capacity of any concentrated or distributed load in the span of the beam between two columns is ignored. Also, the effect of the direct support on the beams provided by the columns is ignored.
- All shear reinforcement is assumed to be perpendicular to the longitudinal reinforcement.
- The effect of any torsion is neglected for the design of shear reinforcement.

The shear reinforcement is designed for each loading combination in the major direction of the beam. In designing the shear reinforcement for a particular beam for a particular loading combination, the following steps are involved (IS 39.2):

- Calculate the design nominal shear stress as

$$\tau_v = \frac{V_u}{A_{cv}},\ A_{cv} = b_w d,\ \text{where} \qquad \text{(IS 39.1)}$$

$$\tau_v \leq \tau_{c,max}, \text{ and} \qquad \text{(IS 39.2.3)}$$

the maximum nominal shear stress, $\tau_{c,max}$ is given in IS Table 14 as follows:

Maximum Shear Stress, $\tau_{c,max}$ (MPa) (IS 39.2.3, IS Table 14)						
Concrete Grade	M15	M20	M25	M30	M35	M40
$\tau_{c,max}$ (MPa)	2.5	2.8	3.1	3.5	3.7	4.0

The maximum nominal shear stress, $\tau_{c,max}$, is computed by the following equation, which matches the IS Table 14 exactly.

$$\tau_{c,\max} = \begin{cases} 2.5 & \text{if} \quad f_{ck} < 15 \\ 2.5 + 0.3\dfrac{f_{ck} - 15}{5} & \text{if} \quad 15 \le f_{ck} < 20 \\ 2.8 + 0.3\dfrac{f_{ck} - 20}{5} & \text{if} \quad 20 \le f_{ck} < 25 \\ 3.1 + 0.4\dfrac{f_{ck} - 25}{5} & \text{if} \quad 25 \le f_{ck} < 30 \\ 3.5 + 0.2\dfrac{f_{ck} - 30}{5} & \text{if} \quad 35 \le f_{ck} < 40 \\ 3.7 + 0.3\dfrac{f_{ck} - 35}{5} & \text{if} \quad 35 \le f_{ck} < 40 \\ 4.0 & \text{if} \quad f_{ck} \ge 40 \end{cases} \qquad \text{(IS 39.2.3)}$$

Calculate the design shear strength of concrete from

$$\tau_{cd} = k\delta\tau_c, \qquad \text{(IS 39.2)}$$

where k is the enhancement factor for the depth of the beam section and is computed by

$$k = 1.6 - 0.002d, \qquad 1.0 \le k \le 1.3. \qquad \text{(IS 39.2.1.1)}$$

The preceding expression represents the table given in IS 39.2.1.1, which is shown below:

The Value of the Enhancement Factor, *k* (IS 39.2.1.1)							
Overall depth of slab, d (mm)	≥300	275	250	225	200	175	≤150
Factor, k	1.00	1.05	1.10	1.15	1.20	1.25	1.30

δ is the enhancement factor for compression and is given by

$$\delta = \begin{cases} 1 + 3\dfrac{P_u}{A_g f_{ck}} \le 1.5 & \text{if} \quad P_u > 0 \qquad \text{Under Compression} \\ 1 & \text{if} \quad P_u \le 0 \qquad \text{Under Tension} \end{cases} \qquad \text{(IS 39.2.2)}$$

δ is always taken as 1, and

τ_c is the basic design shear strength for concrete, which is given by

$$\tau_c = 0.64 \left(\frac{100\, A_s}{bd} \right)^{1/3} \left(\frac{f_{ck}}{25} \right)^{1/4} . \qquad \text{(IS 39.2.1)}$$

The preceding expression tries to represent the IS Table 13 approximately. It should be mentioned that the value of γ_c has already been incorporated in the IS Table 13 (see Note in IS 35.4.2.1). The following limitations are enforced in the determination of the design shear strength as is done in the Table.

$$0.25 \le \frac{100\, A_s}{bd} \le 3, \qquad \text{(IS 39.2.1)}$$

$$f_{ck} \le 40 \text{ MPa (for calculation purpose only).} \qquad \text{(IS 39.2.1)}$$

- The shear reinforcement is computed as follows:

  If $\tau_v \le \tau_{cd} + 0.4$, provide minimum links defined by

  $$\frac{A_{sv}}{s_v} \ge \frac{0.4\, b_w}{0.87\, f_{ys}}, \qquad \text{(IS 39.3 and IS 25.5.1.6)}$$

  else if $\tau_{cd} + 0.4 < \tau_v \le \tau_{c,max}$, provide links given by

  $$\frac{A_{sv}}{s_v} \ge \frac{(\tau_v - \tau_{cd})\, b_w}{0.87\, f_{ys}}, \qquad \text{(IS 39.4)}$$

  else if $\tau_v > \tau_{c,max}$,

  a failure condition is declared. (IS 39.2.3)

  In calculating the shear reinforcement, a limit was imposed on the f_{yv} as

  $$f_{yv} \le 415 \text{ MPa.} \qquad \text{(IS 39.4)}$$

  The maximum of all the calculated A_{sv}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

  The beam shear reinforcement requirements displayed by the program are based purely upon shear strength considerations. Any minimum stirrup re-

quirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

9.5 Slab Design

As is done in conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the limit state of collapse for reinforced concrete (IS 37) as described in the following sections. To learn more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

9.5.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Design flexural reinforcement for the strip

These two steps, described in the sections that follow, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

9.5.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

9.5.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip given the bending moment is identical to the design of rectangular beam sections described earlier. Where the slab properties (depth, etc.) vary over the width of the strip, the program automatically designs slab widths of each property separately for the bending moment they are subjected to before summing up the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

9.5.1.3 Minimum Slab Reinforcement

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limits (IS 25.5.2):

$$A_s =\leq \begin{cases} 0.0015bh & \text{if} \quad f_y < 500MPa \\ 0.0012bh & \text{if} \quad f_y \geq 500MPa \end{cases} \qquad \text{(IS 25.5.2.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (IS 25.5.1.1).

9.5.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the following.

9.5.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of *d/2* from the face of the support (IS 30.6.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (IS 30.6.1).

9.5.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be αM_u and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $(1 - \alpha)\, M_u$ (IS 30.6.2.2) , where

$$\alpha = \frac{1}{1 + (2/3)\sqrt{a_1/a_2}}, \text{ and} \qquad \text{(IS 30.3.3)}$$

where a_1 is the width of the critical section measured in the direction of the span and a_2 is the width of the critical section measured in the direction perpendicular to the span.

9.5.2.3 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as the following.

$$v_c = k_s\, \tau_c, \text{ where} \qquad \text{(IS 30.6.3.1)}$$

$$k_s = 0.5 + \beta c \le 1.0, \qquad \text{(IS 30.6.3.1)}$$

$$\tau_c = 0.25\, \sqrt{f_{ck}}\ , \text{ and} \qquad \text{(IS 30.6.3.1)}$$

β_c = ratio of the minimum to the maximum dimensions of the support section.

9.5.2.4 Determination of Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by SAFE.

References

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